Forward Markets and Oligopoly

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Allaz, Oligopoly, uncertainty and strategic forward transactions, International Journal of Industrial Organization, 1992.

Allaz and Vila, Cournot Competition, Forward Markets and Efficiency, JET, 1993.

We use the simpler model in Allaz and Vila.

Notation:

$i = 1,2$ the two players
z_i = production of player i in the second period
y_i = forward sales in the first period
p_f = forward price
$\nu_i(z_i)$ = cost of producing z_i, we use $\nu_i z_i$. Assume $\nu_1 \leq \nu_2$
$p_s = q(z_1+z_2) = \alpha - z_1 - z_2$

Objective for firm i:

$$\max_{xi} q(z_1+z_2)(z_i-y_i)-\nu_i(z_i)$$

Assuming linear demand and costs

$$0 = \alpha+y_i- 2z_i-z_{-i}-c_i$$

Or

$$z_i = (\alpha+y_i-z_{-i}-c_i)/2$$

Solving simultaneously,

$$z_i = \alpha/3 + (2y_i - y_{-i})/3 - (2\nu_i+\nu_{-i})/3$$

Key property:

x_i increases with f_i and decreases with f_{-i}.

We now construct the closed-loop game of the forward market. To make clear the link with the forward market, we use $z_i(y_i,y_{-i})$ instead of just z_i.

Player i optimization:

$\max_{y_i} (\alpha-z_i(y_i,y_{-i})-z_{-i}(y_{-i},y_i))z_i(y_i,y_{-i})-\nu_i z_i(y_i,y_{-i})$

$0 \leq z_i(y_i,y_{-i})$

Substituting for z_i in the optimization, we get

$\max \{\alpha^2 – (2\nu_i-\nu_{-i})^2-2[\alpha – (2\nu_i-\nu_{-i})]y_{-i} + y_{-i}^2 + [\alpha – (2\nu_i-\nu_{-i}) – y_{-i}]y_i – 2y_i^2\}/9$

subject to $0 \leq \alpha – (2\nu_i-\nu_{-i}) +(2y_i - y_{-i})$

The equilibrium solution is

$y_i = [\alpha – (3\nu_i-2\nu_{-i})]/5$

$z_i = 2[\alpha – (3\nu_i-2\nu_{-i})]/5$

as long as $\alpha \geq (3\nu_i-2\nu_{-i})$

Key result:

Total production higher with futures market

Proof:

From above

$z_i = \alpha/3 + (2y_i - y_{-i})/3 - (2\nu_i + \nu_{-i})/3.$

Adding for i an –i we get

$z_i + z_i = 2\alpha/3 + (y_i + y_{-i})/3 - (\nu_i + \nu_{-i})$

With $y_i=0$ we have the no futures case. However, $y_i>0$ in the futures game.

This can be repeated for more periods of futures markets and as the number of periods approaches infinity, the equilibrium converge to the competitive equilibrium.

Note: This is a prisoner's dilemma game. Simulations with students gets the effect but not the amount projected by Allaz and Vila.

Let's look at lower values of α. Here player 1 uses it lower costs to keep player 2 out of the market.

For $2\nu_2 - \nu_1 < \alpha \leq 3\nu_2 - 2\nu_1$

$z_2 = y_2 = 0$

$z_1 = \alpha - \nu_2$

and

$y_1 = \alpha - 2\nu_2 + \nu_1$

Using the sum of the z's with no futures, we see that total production is higher in this case than with no futures market.

$z_i+z_i = 2\alpha/3 \; - (\nu_i+\nu_{-i}) > \; \alpha - \nu_2$

However, it is not a prisoner's dilemma game as player 1 is better off.

Last point on pure A-V:

If companies have prior positive futures positions before entering the futures market, they will increase their futures position and produce even more. Thus, as the number of periods with futures trading increases, the duopoly equilibrium approaches the competitive equilibrium.

Combining a capacity game with a futures market

First, no load curve and no futures market

The profit from the futures market comes from reducing the other player's production in the following equation for the spot equilibrium.

$z_i = \alpha/3 + (2y_i - y_{-i})/3 - (2\nu_i + \nu_{-i})/3.$

If there is no movement in player z_{-i}'s position from an increase in y_i, then y_i=0 and we have the no futures case. This will drive the analysis in the case with no load curve.

Assume $K_1 + \nu_1 < K_2 + \nu_2$

The optimization is

$\max -K_i x_i + [\alpha - z_i(x_i,x_{-i}) - z_{-i}(x_{-i},x_i) - \nu_i] z_i(x_i,x_{-i})$

$0 \leq z_i(x_i,x_{-i}) \leq x_i$

If an equilibrium exists it is

$z_i = (\alpha - 2\nu_i - 2K_i + \nu_{-i} + K_{-i})/3$

This is the same form as in the no capacity case with the capacity cost added in and no futures market. This is also the same as the solution to the open-loop game. As an open-loop equilibrium it exists and is unique.
Note that

$\lambda_i = K_i$

Thus, capacity must be binding at equilibrium. Note that if $z_i < x_i$, we do not have an equilibrium since player i can improve its position by reducing x_i.

Can player i improve on this solution in the closed-loop game? Maybe. If so, the equilibrium is destroyed.

To see this, the profit function at this equilibrium for player i is

$$(\alpha - x_i - x_{-i}^0 - \nu_i + K_i)x_i$$

$$= [\alpha - x_i^0 - \varepsilon - x_{-i}^0 - \nu_i + (\varepsilon - K_i)/2](x_i^0 + \varepsilon)$$

for $x_i^0 + \varepsilon \geq x_i^0 + K$

Let's look at the duals:

$$\alpha - 2z_i^0 - z_{-i}^0 - \nu_i + \omega_i = \lambda_i \geq 0$$

$$\lambda_i(x_i - z_i) = 0$$

the partial of λ_i with respect to z_{-i} is -1.

That is λ_i must go to 0 before z_i decreases.

We have a numerical example where there is no equilibrium.

Add a futures market to the game

$$\alpha - 2z_i^0 - z_{-i}^0 - \nu_i + y_i = K_i + y_i = \lambda_i$$

The point at which $\lambda_i = 0$ increases with increasing y_i. The same applies to player $-i$.

Solving for ε in the maximization for player i, we get

$$\varepsilon = \alpha - 3x_i^0/2 - x_{-i}^0 - \nu_i - K_i - K_{-i}/2 - y_{-i}/2$$

Note that for profit to increase

$$\varepsilon \geq K + y_{-i}$$

Thus, for sufficiently large y_{-i} player i cannot profit from increasing x_i.

That is, playing the futures market signals that you are serious and will not be moved and makes situations without equilibria have equilibria.

Adding a load curve

With no load curve, capacity must be binding and the value of capacity without a futures position equals the capital cost. (Note the futures game distorts classical duality theory. Therefore, one has to use gradients of the objective function for each load step where capacity is binding.)

With a load curve there are load segments where both players are below capacity, other segments with one at and the other below and still others with both at capacity. This leads to some analytic complexity.

When both are below, this is standard Allaz Vila result. When both are at capacity, the futures positions may be positive to maintain the at-capacity production. In this latter case the at-capacity decisions are consistent with the solutions without a futures market.

Another possibility is that a player would operate below capacity with a futures market. However, this player can use the futures market to establish a position at capacity that leads the other player to lower production. We explain how.

Note, we do not index the load segments to simplify notation.

If $z_i = x_i$,

For player –i

$\alpha - x_i - 2z_{-i} - \nu_{-i} + y_{-i} = 0$

or

$z_{-i} = (\alpha - x_i - \nu_{-i} + y_{-i})/2$

In the futures market the optimization for player –i becomes

$\max\ (\alpha - x_i - z_{-i} - \nu_{-i})z_{-i} = [(\alpha - x_i - \nu_{-i})^2 - y_{-i}^2]/4$

The solution is $y_{-i} = 0$

and

$z_{-i} = (\alpha - x_i - \nu_{-i})/2$

Compare this with z_i from the unconstrained game.

$z_{-i} = 2[\alpha - (3\nu_{-i} - 2\nu_i)]/5$

When $x \leq [\alpha - (2\nu_{-i} - \nu_i)]/2$ and y_i big enough, $y_{-i} = 0$, $z_i = x_i$, and $z_{-i} = (\alpha - x_i - \nu_{-i})/2$

Let's return to the spot equation from the beginning.

$z_i = \alpha/3 + (2y_i - y_{-i})/3 - (2\nu_i+\nu_{-i})/3.$

The higher-profit, no futures equilibrium is

$\alpha/3 - (2\nu_i+\nu_{-i})/3.$

Whenever $\alpha/3 - (2\nu_i+\nu_{-i})/3 < x_i$ and $z_i = x_i$, the value of another unit of capacity is negative.

Since the values of capacity have to add up to the cost of capacity, the λ's that are greater than 0 have to sum to a total $> K_i$. This implies lower capacity x_i.

$\omega\ \lambda\ \alpha \leq \geq \varepsilon\ \nu$