INVESTMENT DYNAMICS IN ELECTRICITY MARKETS

Alfredo Garcia, University of Virginia

joint work with

Ennio Stacchetti, New York University

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MOTIVATION

- We study “resource adequacy” as an endogenous feature in a dynamic game model of investments in electricity markets.
- In the early days of restructuring, the conventional wisdom on the subject seemed to be that a perfectly competitive market would have a self regulating ability to induce the socially optimal level of reliability in the long run, *provided* some regulatory distortions inherited from the old regulatory regimes were eliminated.

- According to this conventional wisdom, one of the most prominent regulatory distortions is the existence of *price caps* that limit the amount of scarcity rents that peaking plants may accrue in situations with little excess capacity in the market. This is informally referred to as the “missing money” problem (see Joskow (2006)).
- In this paper, we construct a model of strategic investment dynamics. In our equilibria, a policy of increasing price caps *has no effect* in the long-run levels of excess capacity

MODEL SETUP

- Assume there are 2 firms with constant marginal cost of production $c > 0$ up to its current capacity.
- A price cap $\bar{p} > c$ is stipulated by the regulatory commission. For later use, denote by $m = \bar{p} - c$ the maximum markup allowed by the commission.
- Let $K^t = (K_1^t, K_2^t)$ be the firms' capacities and D^t be the inelastic demand in period t.

- Firm i has u_i units and K_i^t is equally divided among its units, $s_i^t = \frac{K_i^t}{u_i}$
- At each period, firms simultaneously submit bids for each unit $b_i \in [0, \bar{p}]^{u_i}$
- From lowest to highest, the units are dispatched until their combined capacities is greater than or equal to D^t. Ties are broken randomly.
- The last unit dispatched, say j, is called the *marginal firm* and its bid sets the *spot price* for the market in period t.

THE PRICE AUCTION WITH DISCRETE SUPPLY FUNCTIONS

Assume that the current capacities are $K = (K_1, K_2)$ and that current demand is D. We restrict our presentation to two separate cases (see paper for details).

Case 1: $K_1 + K_2 \leq D$. Here there is a unique equilibrium price equal to $\bar{p}$. The corresponding revenues are

$$(R_1, R_2) = (mK_1, mK_2).$$

Case 2: $K_1 + K_2 > D$. In this case there are multiple equilibria for the one-shot auction game. In one pure-strategy equilibrium $(b_1, b_2) = (c, \overline{p})$ and the corresponding revenues are

$$(R_1, R_2) \;=\; (mK_1, m(D - K_1))$$

Another pure-strategy equilibrium is $(p_1, p_2) = (\overline{p}, c)$ with payoffs

$$(R_1, R_2) \;=\; (m(D - K_2), mK_2)$$

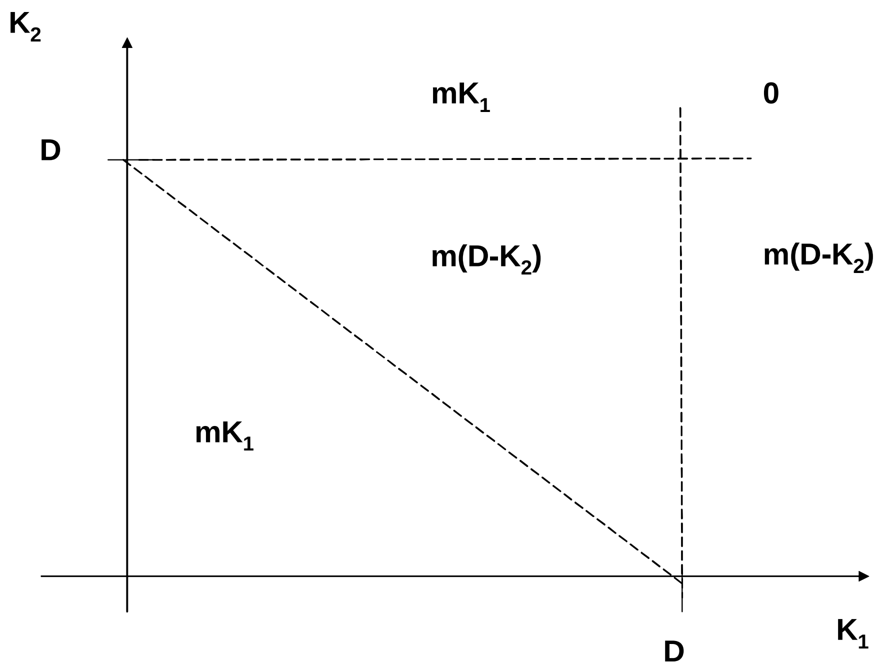
There also is a continuum of mixed strategy equilibria with equilibrium payoffs:

$$R = (m[(1-\bar{\varphi}_2)(D-K_2)+\bar{\varphi}_2 K_1], m[(1-\bar{\varphi}_1)(D-K_1)+\bar{\varphi}_1 K_2])$$

where the probabilities $(\bar{\varphi}_1, \bar{\varphi}_2)$ satisfy $\bar{\varphi}_i \in [0,1)$, $i = 1,2$, and $\bar{\varphi}_1\bar{\varphi}_2 = 0$. We select the equilibrium in which $\bar{\varphi}_1 = \bar{\varphi}_2 = 0$ (i.e. the unique symmetric equilibrium). This equilibrium is the “most competitive”.

- Other cases are described in the paper.
- To summarize, the equilibrium payoff function for firm 1 under the selected equilibrium strategy is depicted in Figure 1.

Figure 1: Equilibrium Payoffs for Firm 1



MORE ON THE EQUILIBRIUM SELECTED

Let $E = K_1 + K_1 - D$ and $a = D/E - 1$.

The expected price in the mixed-strategy equilibrium is

$$\hat{p} = \int_c^{\overline{p}} pa\frac{(p-c)^{a-1}}{m^a}dp = (\overline{p} - c)\left[1 - \frac{E}{D}\right] + c.$$

When $E = 0$, $\hat{p} = \bar{p}$. Also as $E \to D$, $\hat{p} \to c$.

A SIMPLE TWO-PERIOD INVESTMENT GAME

- Consider the following simple two-period investment game.
- In period 0, two firms having zero initial capacity are to decide how much capacity, say (K_1, K_2), to install.
- Demand D is equal to $1 + g$ with probability θ and 1 with probability $1 - \theta$, where $g \in (0, 1)$
- In period 1, the uncertainty over demand is resolved and price competition results in an infinite stream of net revenues $R_i(K_1, K_2), i \in \{1, 2\}$

- We first note that firm 1 will never invest in excess of $1+g-K_2$. This follows from the fact that net revenues are independent of K_1 whenever $K_1 > 1+g-K_2$.
- Suppose firm 1 chooses a capacity level $K_1 \in [1-K_2, 1+g-K_1]$ and $g < K_2 < 1$. Assuming $\kappa > 0$ is the constant marginal cost of investments, the firm's discounted profit is:

$$V_1(K_1, K_2) = \frac{\beta}{1-\beta}[\theta m K_1 + (1-\theta)m(1-K_2)] - \kappa K_1$$

where $\beta = (1+\rho)^{-1}$ is the discount factor.

- Firm 1's best reply is $1+g-K_2$ if

$$\frac{\partial V_1}{\partial K_1}(K_1, K_2) = \frac{\beta\theta m}{1-\beta} - \kappa > 0$$

Or equivalently,

$$\frac{m}{\kappa} > \frac{1-\beta}{\beta\theta} = \frac{\rho}{\theta}$$

- Provided this condition holds, there exists a symmetric equilibrium in which

$$(K_1^*, K_2^*) = (\frac{1+g}{2}, \frac{1+g}{2})$$

- Note that while this equilibrium guarantees "security of supply" or "adequate investment" it becomes less plausible as the high demand scenario becomes less likely (i.e. as $\theta \to 0$).

THE INVESTMENT GAME

- At the end of the period, the firms simultaneously choose capacity investments $Y_i^t \geq 0$, $i = 1, \ldots, n$.
- The constant marginal cost of investment is $\kappa > 0$. Hence, firm i's net profit for period t is

$$\pi_i^t = R_i^t - \kappa Y_i^t$$

and its capacity for next period becomes $K_i^{t+1} = K_i^t + Y_i^t$.

- Demand grows as follows:

$$\frac{D^{t+1}}{D^t} = \begin{cases} 1 & \text{with probability } 1-\theta \\ \\ 1+g & \text{with probability } \theta \end{cases}$$

where $g > 0$ and $\theta \in [0, 1]$.

- We now arbitrarily fix the behavior of the firms in each auction game to be the bidding equilibrium strategy selected.
- By exogenously fixing the behavior of the firms at the auctions, we obtain a residual dynamic game where the firms only choose investments.
- We restrict attention to investment strategies where the decisions of the firms in period t depend exclusively on the current capacity stock K^t and demand D^t.

VALUE FUNCTION

For a given investment strategy combination $Y(K, D)$ the value function can be defined as follows:

$$V_i^Y(K, D) = E\left[\sum_{t \geq 0} \beta^t (R_i^t - \kappa Y_i^t)\right]$$

where $\beta = \frac{1}{1+r} \in (0, 1)$ is the discount factor and (K, D) are the given initial conditions on capacity stock and demand.

INVESTMENT EQUILIBRIUM

An investment function strategy combination $Y(K, D)$ is an equilibrium iff for all i, $\hat{Y}_i$ and initial condition (K, D)

$$V_i^Y(K, D) \geq V_i^{\hat{Y}}(K, D)$$

where $\hat{Y} = (\hat{Y}_i, Y_{-i})$. In words, firm i does not have an incentive to switch to investment strategy $\hat{Y}_i$ when all other firms are investing according to Y_{-i}.

“BASE-STOCK” INVESTMENT STRATEGIES

Given initial condition (K, D) we are interested in strategy combinations in which each player’s investment is

$$Y^*(K,D) = \begin{cases} \frac{1}{2}[(1+g)D - K_1 - K_2] & \text{if } K_1 + K_2 < (1+g)D \\ \\ 0 & \text{if } K_1 + K_2 \geq (1+g)D \end{cases}$$

- In Figure 2 and 3 we show an example of the application of this strategy when $D = 1$.

Figure 2: With probability $1 - \theta$

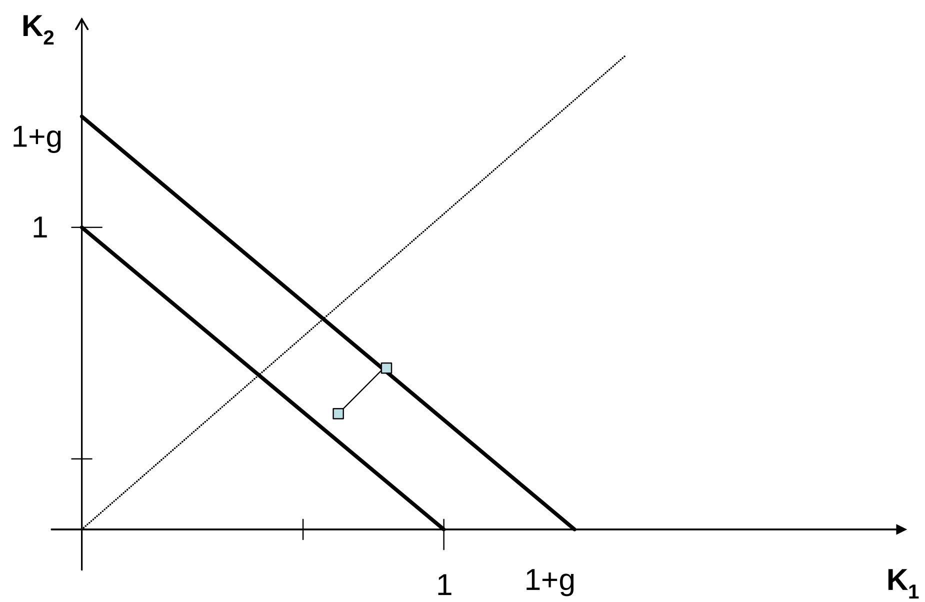
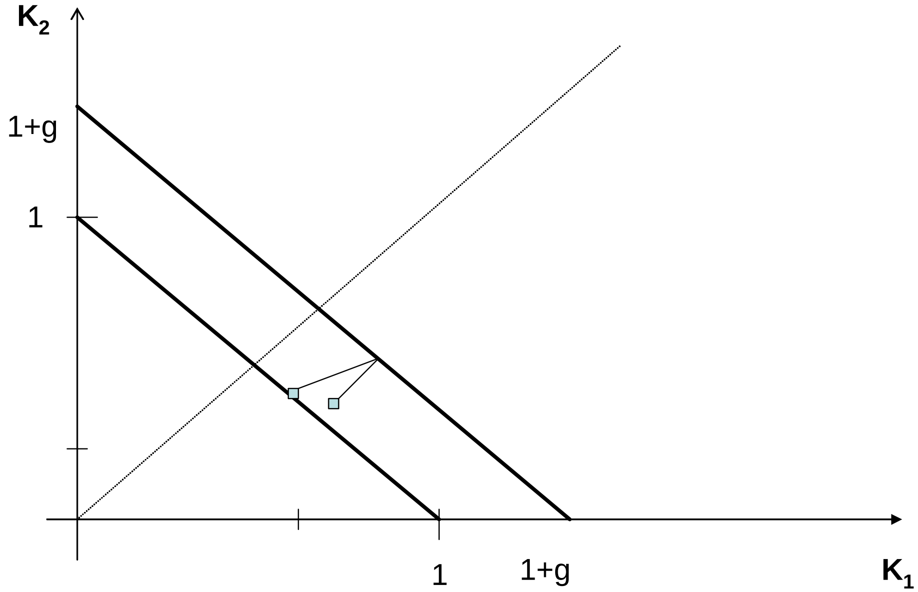


Figure 3: With probability θ



NO “SECURITY OF SUPPLY”

Given initial condition (K, D) we are interested in strategy combinations in which each player’s investment is

$$\hat{Y}(K,D) = \begin{cases} \frac{1}{2}[D - K_1 - K_2] & \text{if } K_1 + K_2 < D \\ 0 & \text{if } K_1 + K_2 \geq D \end{cases}$$

Theorem 1: *Let* $\eta = \frac{\beta\theta}{1-\beta(1-\theta)}$. *Assuming*:

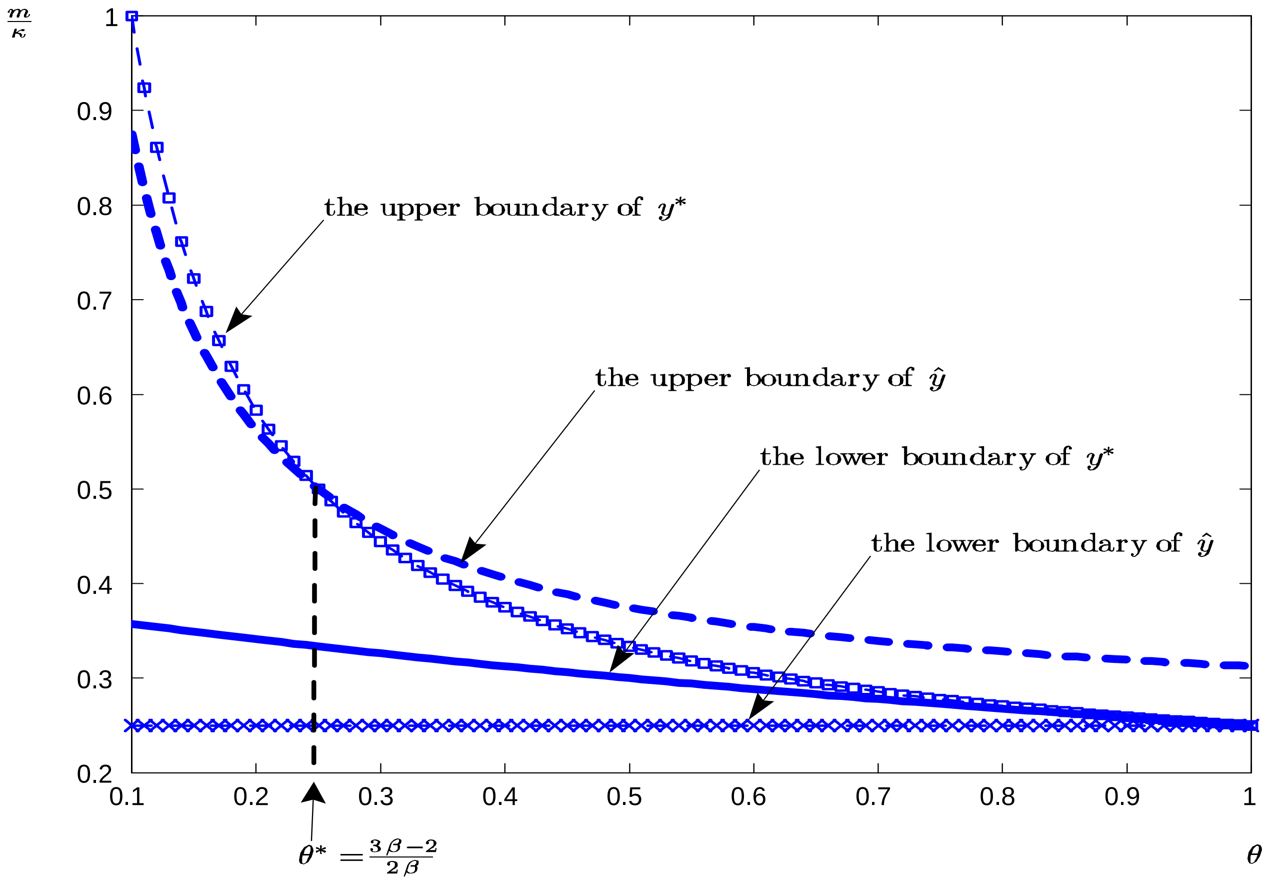
$$\frac{(2-\beta)(1-\beta)}{\beta[p(2-\beta)+(1-p)\beta]} < \frac{m}{\kappa} < \frac{1-\beta}{\beta\eta}$$

the strategy combination $Y^*(K,D)$ *is an equilibrium.*

Theorem 2: *Assuming*:

$$r = \frac{(1-\beta)}{\beta} < \frac{m}{\kappa} < \frac{(2-\eta)(1-\beta)}{\eta(2-\beta)}$$

the strategy combination $\hat{Y}(K,D)$ *is an equilibrium.*



$\frac{m}{\kappa}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
θ
the upper boundary of y^*
the upper boundary of $\hat{y}$
the lower boundary of y^*
the lower boundary of $\hat{y}$
$\theta^* = \frac{3\beta-2}{2\beta}$

Figure 4

- The lower bound on $\frac{m}{\kappa}$ is determined by the incentive to *under-invest* and is decreasing in p. For example when $\theta \to 1$, the condition requires (in the case of Y^*)

$$r < \frac{m}{\kappa}$$

- The upper bound on $\frac{m}{\kappa}$ is determined by the incentive to *over-invest* and is also decreasing in p. For instance, when $\theta \to 0$, the condition requires (in the case of Y^*)

$$r + 2r^2 < \frac{m}{\kappa} < \infty$$

EXCESS CAPACITY

In the long run, excess capacity (or shortage) is:

$$\begin{cases} g & \text{with probability } 1-\theta \\ \\ 0 & \text{with probability } \theta \end{cases}$$

and the average level of excess capacity (or shortage) is

$$E^* = (1-\theta)g > 0$$

Now, let's compare the feasible regions for each one of these equilibrium strategies (see Figure 4).

- It can be shown that for low values of p, the equilibrium $\hat{Y}(K, D)$ Pareto-dominates $Y^*(K, D)$.
- Note that under $\hat{Y}(K, D)$, in the long run, excess capacity (or shortage) is:

$$\begin{cases} 0 & \text{with probability } 1-\theta \\ \\ -g & \text{with probability } \theta \end{cases}$$

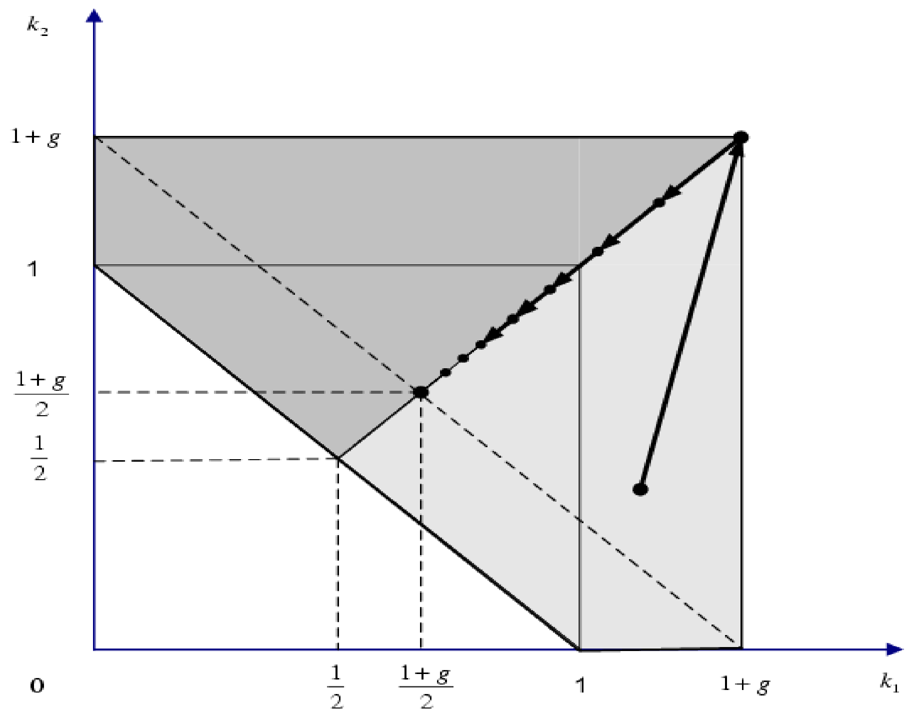
and the average level of excess capacity (or shortage) is $\hat{E} = -\theta g < 0$

EQUILIBRIUM WITH EXCESS CAPACITY

We concentrate on the case $\theta = 1$. Consider the strategy

$$Y_1^*(K,D) = \begin{cases} \frac{1}{2}(D(1+g)-K_1-K_2) & K_1+K_2 \leq D \text{ or } (K_1,K_2) \in A \\ D(1+g)-K_1 & K_1+K_2 > D, K_i \leq D(1+g) \\ & K_1 \neq K_2 \\ \frac{Dg}{2} & K_1 = K_2, K_i \in [\frac{D}{2}, D] \\ 0 & \text{in all other cases} \end{cases}$$

where $A = \{(K_1,K_2) : K_1K_2 = 0 \text{ and } K_i \in (D, D(1+g)]\}$



Equilibrium with Excess Capacity

Theorem 3: *Assuming*:

$$\frac{m}{\kappa} \geq \frac{1-\beta}{\beta}[2\frac{1-\beta}{\beta g} - 1] = \rho[2\frac{\rho}{g} - 1]$$

the strategy Y^* *is a Markov Perfect Equilibrium.*

IMPLICATIONS FOR REGULATORY POLICY

- Our results indicate that there are investment equilibrium strategies under which the firms tend relatively small levels of excess capacity.
- In some cases, capacity is often insufficient to cover demand and rationing occurs.
- The long run average level of excess capacity *is not affected* by the value of the price cap.