Ence 627

Decision Analysis for Engineering

Project Portfolio Selection:

"Optimal Budgeting of Projects Under Uncertainty"

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Overview

"Optimal Budgeting of Projects Under Uncertainty"

- 4 Projects to be allocated in 3 years
- Projects' costs are uncertain
- No more than 2 projects can be selected per year
- Initial budget is also uncertain
- The policy of the budget use says that as much as the budget should be used (minimize residual)

How do I decide what project to fund and when?

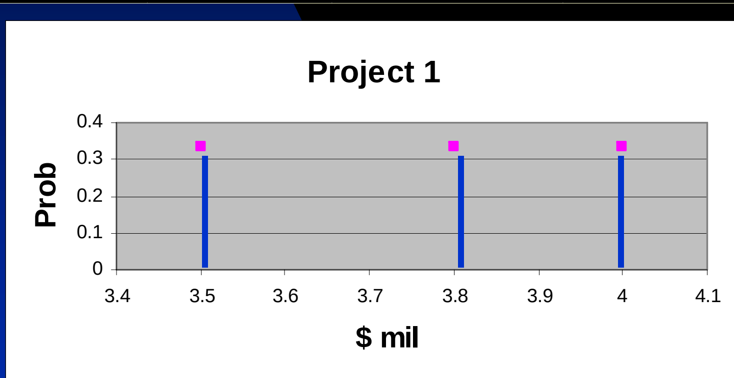
Stochastic Dynamic Programming:

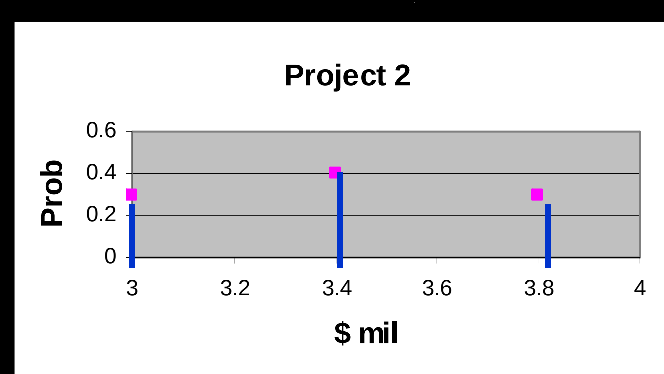
- SDP is a framework for modeling optimization problems that involve uncertainty.
- The goal is to find a policy which is feasible for all (or almost all) the possible data instances and maximizes/minimizes the expectation of some function of the decision and the random variables.
- SDP takes advantage of the fact that probability distributions are known or can be estimated.
- Recourse: ability to take corrective action after a random event has taken place

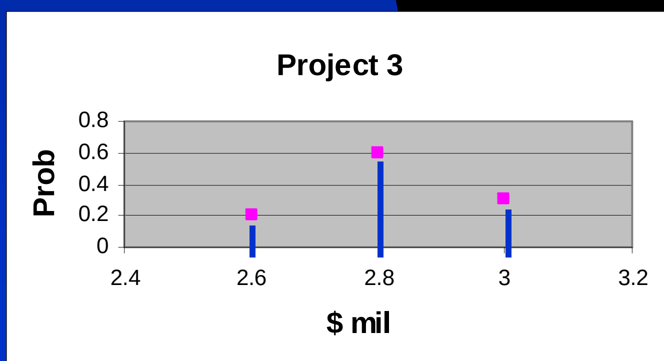
Data

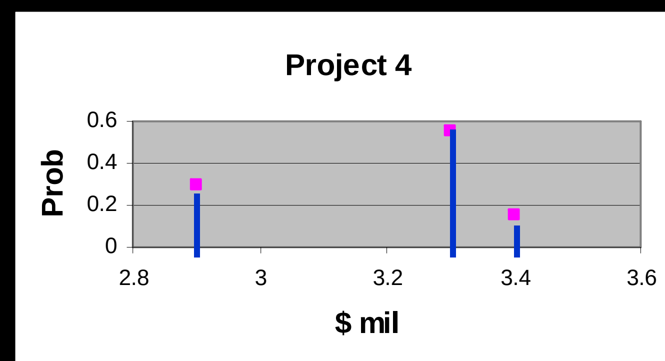
Projects' Cost

	C(i, j)						
	C(i, 1)	P(i , 1)	C(i, 2)	P(i , 2)	C(i, 3)	P(i , 3)	
Project (i)	Minimun Cost	Probab	Normal Cost	Probab	Maximum Cost	Probab	*Expected Cost*
1	3.5	0.33	3.8	0.33	4	0.33	3.80
2	3	0.3	3.4	0.4	3.8	0.3	3.40
3	2.6	0.2	2.8	0.6	3	0.2	2.80
4	2.9	0.3	3.3	0.55	3.4	0.15	3.20









* Initial budget does not exceed $14 mil

SDP elements

- Stages

  time: 1^{st}, 2^{nd}, 3^{rd} year

- States

  Budget left and project selected in a previous stage

  Example: (10.1, 1, 0, 0, 0)

- Actions

  Projects chosen in the period

SDP general formulation

$$f_t(i) = \min_a \left\{ \left(\text{Expected reward during stage t} \middle| i, a\right) + \sum_j p\left(j \middle| i, a, t\right) f_{t+1}(j) \right\}$$

- *$f_t(i)$* minimum expected reward that can be earned during stages *t, t+1…. ,end of the problem* given that the state at the beginning of stage *t* is *i*
- The minimum is taken over all actions *a* that are feasible when the state at the beginning of state *t* is *i*
- *P(j / i,a,t)* is the probability that the next period state will be *j*, given that the current *(stage t)* state is *i* and action *a* is chosen.
- The summation represents the expected reward from stage *t+1* to the end of the problem
- By choosing *a* to minimize the right hand side of eq. , we are choosing *a* to minimize the expected reward earned from stage t to the end of the problem

* In this case the expected reward is defined as the expected deviation of the total cost from the budget left, and this is what we want to minimize, we call it "residual".

Resources

- Excel: data storage and interface
- Visual Basic & Excel Macros:

  Iterations, calculations and generation of files used by MPL
- Optimax object: Add-in that allows to connect Excel with MPL
- MPL optimization software: using linear programming gives the optimal project selection to minimize the expect residual

Calculations

- Generation of $f_3(x) = r_3(x)$

Stage 3							
STATE						Action	**Expected residual**
Budget left	Project 1	Project 2	Project 3	Project 4		Projects Choosen	**Budget - E(Cost)**
8	1	0	0	0		**4 2**	4.7
8	0	1	0	0		**4 1**	4.7
8	0	0	1	0		**2 1**	4.6
8	0	0	0	1		**2 1**	4.6
8	1	1	0	0		**4 3**	4.7
8	1	0	1	0		**4 2**	4.7
8	1	0	0	1		**3 2**	4.6
8	0	1	1	0		**4 1**	4.7
8	0	1	0	1		**3 1**	4.6
8	0	0	1	1		**2 1**	4.6
8	1	1	1	0		**4**	4.8
8	1	1	0	1		**3**	5.1
8	1	0	1	1		**2**	4.6
8	0	1	1	1		**1**	4.2
7.9	1	0	0	0		**4 2**	4.6
7.9	0	1	0	0		**4 1**	4.6
7.9	0	0	1	0		**2 1**	4.5
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
0	1	0	0	1			0
0	0	1	1	0			0
0	0	1	0	1			0
0	0	0	1	1			0
0	1	1	1	0			0
0	1	1	0	1			0
0	1	0	1	1			0
0	0	1	1	1			0

Generation of $r_2(x)$

Stage 2											
STATE						Action	Expected	State after selection			
Budget left	Project 1	Project 2	Project 3	Project 4		Projects Choosen	Budget - E(Cost)	Project 1	Project 2	Project 3	Project 4
10	1	0	0	0		**4 2**	6.7	1	1	0	1
10	0	1	0	0		**4 1**	6.7	1	1	0	1
10	0	0	1	0		**2 1**	6.6	1	1	1	0
10	0	0	0	1		**2 1**	6.6	1	1	0	1
10	1	1	0	0		**4 3**	6.7	1	1	1	1
10	1	0	1	0		**4 2**	6.7	1	1	1	1
10	1	0	0	1		**3 2**	6.6	1	1	1	1
10	0	1	1	0		**4 1**	6.7	1	1	1	1
10	0	1	0	1		**3 1**	6.6	1	1	1	1
10	0	0	1	1		**2 1**	6.6	1	1	1	1
9.9	1	0	0	0		**4 2**	6.6	1	1	0	1
9.9	0	1	0	0		**4 1**	6.6	1	1	0	1
9.9	0	0	1	0		**2 1**	6.5	1	1	1	0
.	.	.	.	.	.	.	.				
.	.	.	.	.	.	.	.				
0	1	1	0	0			0	1	1	0	0
0	1	0	1	0			0	1	0	1	0
0	1	0	0	1			0	1	0	0	1
0	0	1	1	0			0	0	1	1	0
0	0	1	0	1			0	0	1	0	1
0	0	0	1	1			0	0	0	1	1

Generation of r3(x)

Stage 1												
STATE					#Projects	Action	**Expected**	**State after selection**			
Budget left	Project 1	Project 2	Project 3	Project 4	allowed	Projects Choosen	**Budget - E(Cost)**	Project 1	Project 2	Project 3	Project 4
14	0	0	0	0	1	**1**	10.2	1	0	0	0
14	0	0	0	0	2	**2 1**	10.6	1	1	0	0
13.9	0	0	0	0	1	**1**	10.1	1	0	0	0
13.9	0	0	0	0	2	**2 1**	10.5	1	1	0	0
13.8	0	0	0	0	1	**1**	10	1	0	0	0
13.8	0	0	0	0	2	**2 1**	10.4	1	1	0	0
13.7	0	0	0	0	1	**1**	9.9	1	0	0	0
13.7	0	0	0	0	2	**2 1**	10.3	1	1	0	0
13.6	0	0	0	0	1	**1**	9.8	1	0	0	0
13.6	0	0	0	0	2	**2 1**	10.2	1	1	0	0
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
0.2	0	0	0	0	1		0.2	0	0	0	0
0.2	0	0	0	0	2		0.2	0	0	0	0
0.1	0	0	0	0	1		0.1	0	0	0	0
0.1	0	0	0	0	2		0.1	0	0	0	0
0	0	0	0	0	1		0	0	0	0	0
0	0	0	0	0	2		0	0	0	0	0

Next Steps

- Applying SDP formulation:

for t =3

$$f_3(x) = r_3(x)$$

for t = 1 ,2

$$f_t(i) = \min_a \{ r_t(a_t) + f_{t+1}(i - a_t) \}$$

Recursive relationship

Residual for stage t if action a is taken

Residual from other stages

Calculation example

Stage: 2

State i:

STATE				
Budget left	Project 1	Project 2	Project 3	Project 4
10	1	0	0	0

Possible a: {(10,9.9,….0),1,0,0,0}

For each a $r_2(a_2)$

STATE						Action	Expected	State after selection			
Budget left	Project 1	Project 2	Project 3	Project 4		Projects Choosen	**Budget - E(Cost)**	Project 1	Project 2	Project 3	Project 4
10	1	0	0	0		**4 2**	6.7	1	1	0	1
9.9	1	0	0	0		**4 2**	6.6	1	1	0	1
9.8	1	0	0	0		**4 2**	6.5	1	1	0	1
9.7	1	0	0	0		**4 2**	6.4	1	1	0	1
9.6	1	0	0	0		**4 2**	6.3	1	1	0	1
9.5	1	0	0	0		**4 2**	6.2	1	1	0	1
9.4	1	0	0	0		**4 2**	6.1	1	1	0	1
9.3	1	0	0	0		**4 2**	6	1	1	0	1
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
0.7	1	0	0	0			0.7	1	0	0	0
0.6	1	0	0	0			0.6	1	0	0	0
0.5	1	0	0	0			0.5	1	0	0	0
0.4	1	0	0	0			0.4	1	0	0	0
0.3	1	0	0	0			0.3	1	0	0	0
0.2	1	0	0	0			0.2	1	0	0	0
0.1	1	0	0	0			0.1	1	0	0	0
0	1	0	0	0			0	1	0	0	0

Get $f_3(i-a_2)$

Stage 3							
STATE						Action	Expected
Budget left	Project 1	Project 2	Project 3	Project 4		Projects Choosen	Budget - E(Cost)
6.7	1	1	0	1		**3**	3.8
6.6	1	1	0	1		**3**	3.7
6.5	1	1	0	1		**3**	3.6
6.4	1	1	0	1		**3**	3.5
6.3	1	1	0	1		**3**	3.4
6.2	1	1	0	1		**3**	3.3
6.1	1	1	0	1		**3**	3.2
6	1	1	0	1		**3**	3.1
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
0.7	1	0	0	0			0.7
0.6	1	0	0	0			0.6
0.5	1	0	0	0			0.5
0.4	1	0	0	0			0.4
0.3	1	0	0	0			0.3
0.2	1	0	0	0			0.2
0.1	1	0	0	0			0.1
0	1	0	0	0			0

Applying formulation

$$f_2(10) = \min_a \{r_2(a_2) + f_3(i - a_2)\}$$

$r_2(a_2)$	$f_3(1-a_2)$	$f_2(a_2)$
6.7	3.8	10.5
6.6	3.7	10.3
6.5	3.6	10.1
6.4	3.5	9.9
6.3	3.4	9.7
6.2	3.3	9.5
6.1	3.2	9.3
6	3.1	9.1
.	.	.
.	.	.

$Min_a = f_2(10)$

Results

Example: budget available=$14 mil

Stage 1							
	STATE				#Projects		
Budget left	Project 1	Project 2	Project 3	Project 4	allowed	**E (r)**	Projects Choosen
14	0	0	0	0	1	6.8	**1**
14	0	0	0	0	2	6.9	**1**
13.9	0	0	0	0	1	6.7	**1**
13.9	0	0	0	0	2	6.8	**1**
13.8	0	0	0	0	1	6.6	**1**
13.8	0	0	0	0	2	6.7	**1**
13.7	0	0	0	0	1	6.5	**1**
13.7	0	0	0	0	2	6.6	**1**
13.6	0	0	0	0	1	6.4	**1**
13.6	0	0	0	0	2	6.5	**1**
13.5	0	0	0	0	1	6.3	**1**
13.5	0	0	0	0	2	6.4	**1**
13.4	0	0	0	0	1	6.2	**1**
13.4	0	0	0	0	2	6.2	**3 2**
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
0.3	0	0	0	0	1		
0.3	0	0	0	0	2		
0.2	0	0	0	0	1		
0.2	0	0	0	0	2		
0.1	0	0	0	0	1		
0.1	0	0	0	0	2		
0	0	0	0	0	1		
0	0	0	0	0	2		

Results

Actual cost of project 1 = $4.1 mill

Budget left = $9.9 mill

State: (9.9,1,0,0,0)

Stage 2						
STATE						**projects**
Budget left	Project 1	Project 2	Project 3	Project 4	$f_{(2)}$	**chosen**
10	1	0	0	0	3.3	**4 3**
10	0	1	0	0	2.9	**4 3**
10	0	0	1	0	2.8	**1**
10	0	0	0	1	2.8	**1**
10	1	1	0	0	3.9	**4**
10	1	0	1	0	3.4	**2**
10	1	0	0	1	3.7	**2**
10	0	1	1	0	3	**1**
10	0	1	0	1	3.3	**1**
10	0	0	1	1	2.8	**1**
9.9	1	0	0	0	3.2	**4 3**
9.9	0	1	0	0	2.9	**4 3**
9.9	0	0	1	0	2.7	**1**
9.9	0	0	0	1	2.7	**1**
.	.	.	.	.	.	.
.	.	.	.	.	.	.
0.1	0	1	1	0	0.1	
0.1	0	1	0	1	0.1	
0.1	0	0	1	1	0.1	

Results

Actual cost of project 1 = $ 4.1 mill

Actual cost of project 3 = $ 3 mill

Actual cost of project 4 = $ 3.3 mill

Total = $ 10.4 mill

Budget left = $3.6 mill

State: (3.6,1,0,1,1)

Stage 3							
		STATE				Action	**Expected**
Budget left	Project 1	Project 2	Project 3	Project 4		Projects Choosen	**Budget - E(Cost)**
3.7	0	1	1	1			3.4
3.6	1	0	0	0		**2**	0.4
3.6	0	1	0	0		**4**	0.4
3.6	0	0	1	0		**2**	0.4
3.6	0	0	0	1		**2**	0.4
3.6	1	1	0	0		**4**	0.4
3.6	1	0	1	0		**2**	0.4
3.6	1	0	0	1		**2**	0.4
3.6	0	1	1	0		**4**	0.4
3.6	0	1	0	1		**3**	0.7
3.6	0	0	1	1		**2**	0.4
3.6	1	1	1	0		**4**	0.4
3.6	1	1	0	1		**3**	0.7
3.6	1	0	1	1		**2**	0.4
3.6	0	1	1	1			3.6
3.5	1	0	0	0		**4**	0.3

Model statistics

Possible states: 2426

Iterations for possible scenarios: around 300’000,000

Time to run the program 1h15

Future work

- Increase the number of projects
- Multi-objective optimization: cost & resources
- Improve program functionality

Questions?