Backcalculation Analysis of Pavement-layer Moduli Using Pattern Search Algorithms

Project Report for ENCE 724

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Backcalculation Analysis of Pavement-layer Moduli Using Pattern Search Algorithms

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1. Introduction

- Evaluation of existing in-service pavement is important in determination of pavement construction quality and assessment of rehabilitation needs.

- two possible methods for evaluating the pavement:

(1)lab testing

(2)nondestructive testing (NDT)

falling weight deflectometer (FWD)

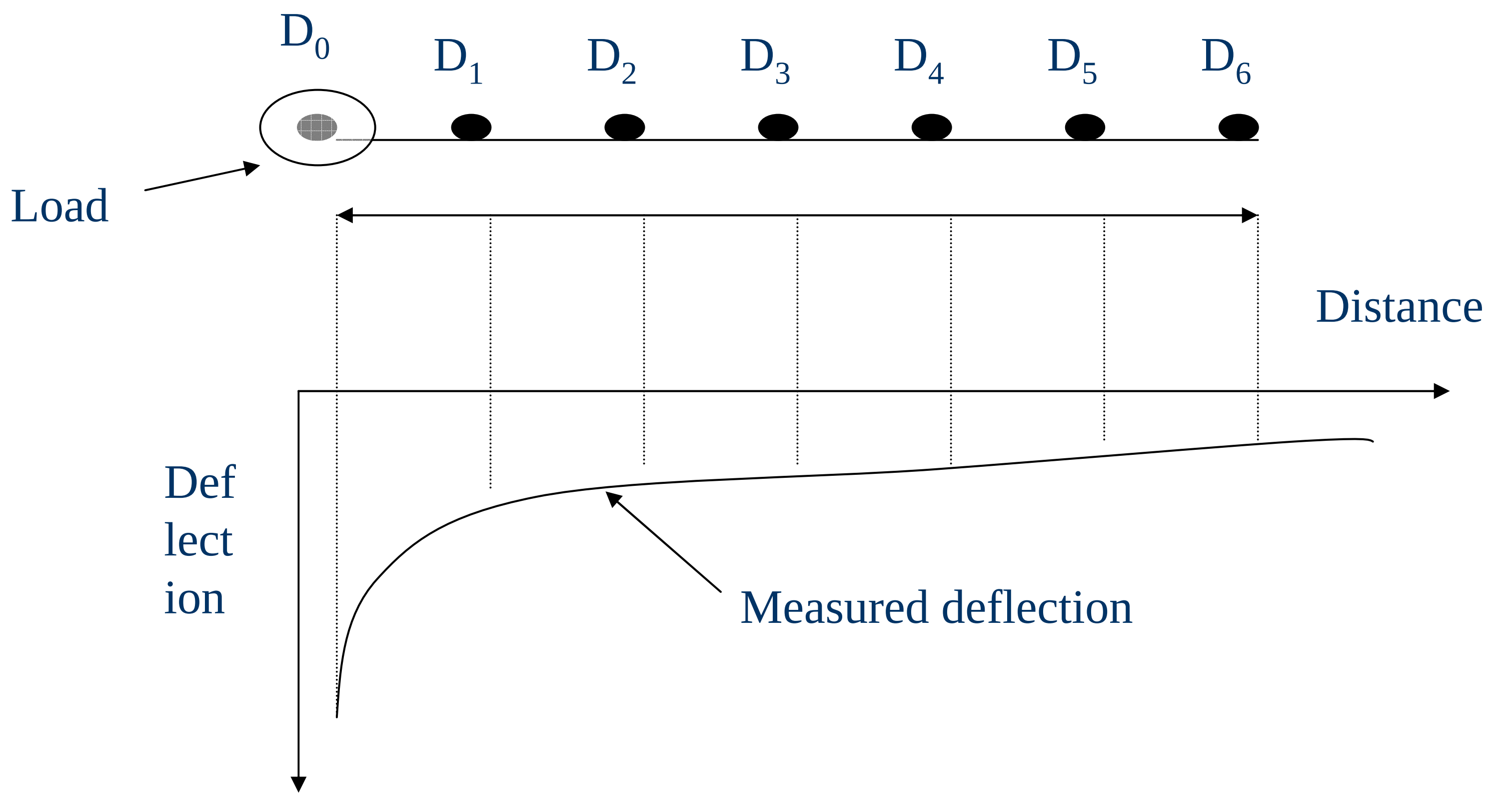
2. Overview of the Project
Typical Flexible Pavement Structure

Asphalt Concrete

Base

Subbase/Subgrade

2. Overview of the Project
Falling Weight Deflectometer(FWD) Test



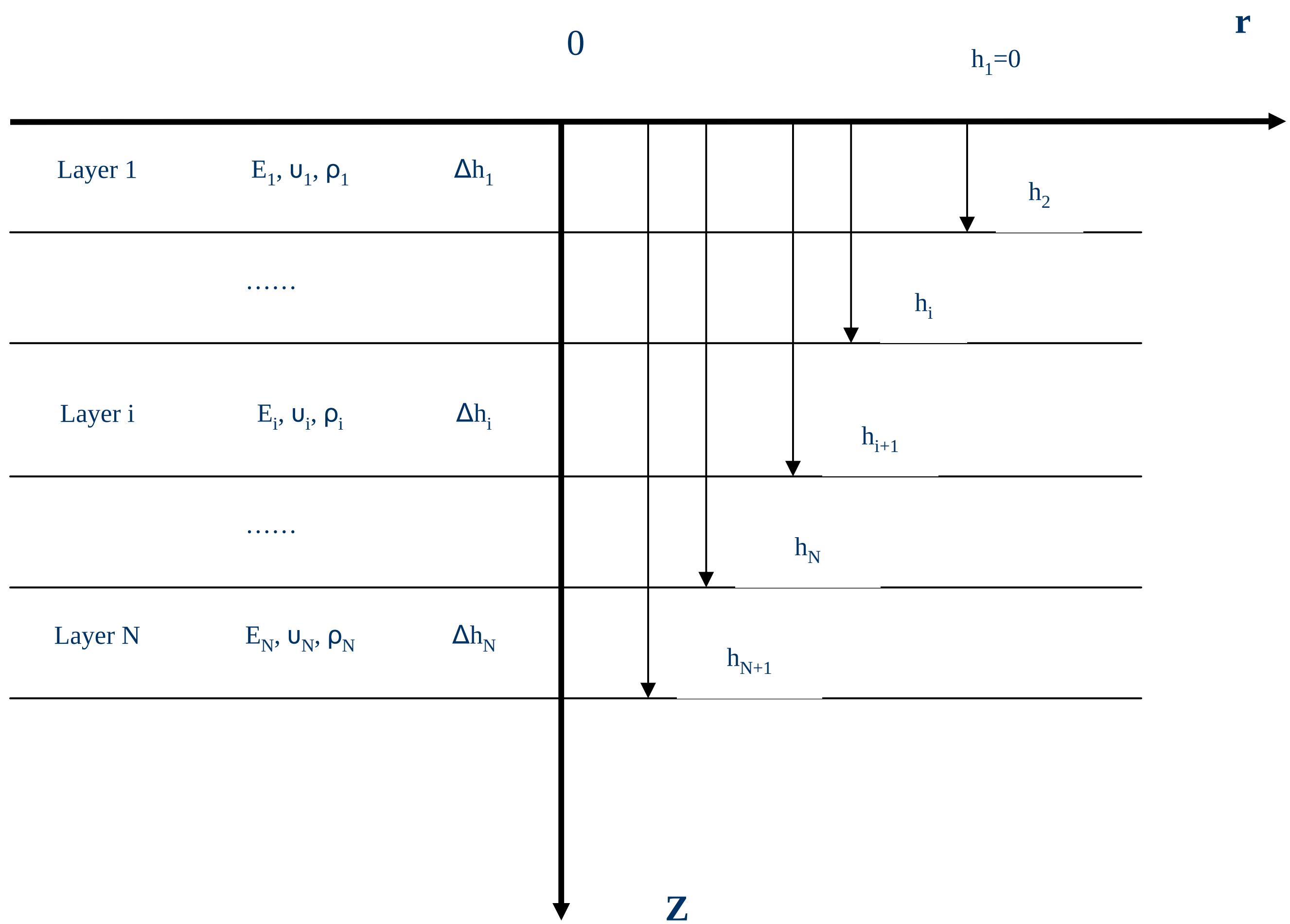
Schematic of Deflection Basin, and Loading, Sensors Configuration

3. Objective of the Project

- To develop an effective method to backcalculate the layer moduli of pavement from the FWD deflection data.

4. Pavement Model

Basic Model



4. Pavement Model
Governing Equation

$$(c_d^2 - c_s^2)\frac{\partial \Delta}{\partial r} + c_s^2\left(\nabla^2 u - \frac{u}{r^2}\right) - \ddot{u} = 0 \qquad (c_d^2 - c_s^2)\frac{\partial \Delta}{\partial z} + c_s^2 \nabla^2 w - \ddot{w} = 0$$

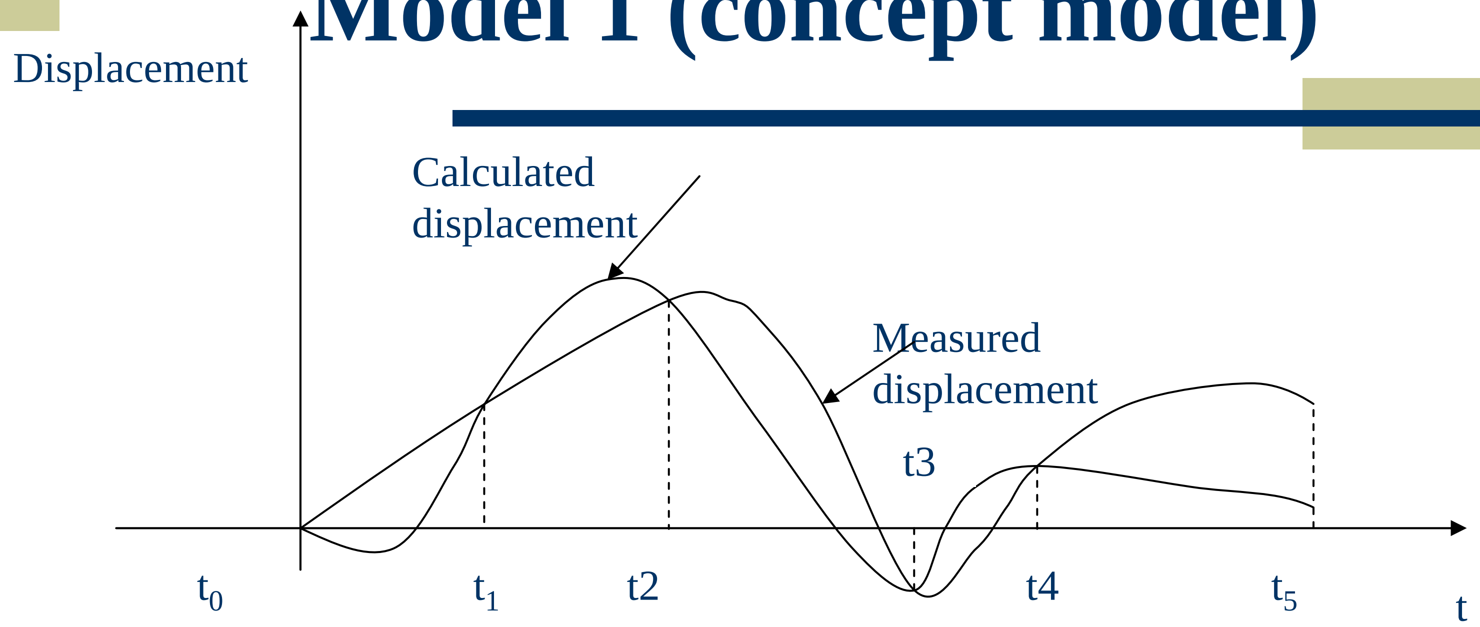
- Where $u = u(r,z,t)$:displacements of the i th layer along the r ;
- $w = w(r,z,t)$: displacements of the i th layer z direction;
- dots indicate differentiation with respect to time t ;
- c_d : dilatational wave velocity
- c_s : shear wave velocity

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \qquad \Delta = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}$$

4. Pavement Model
Solution for Multiple layer Model with Rock bed.

$$u(r,Z,t)=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\int_{0}^{\infty}\xi J_1(\xi r)\left[\psi_1(Z)\right]\left[\bar{\tilde{\tau}}_{rz}(h_1),\bar{\tilde{\sigma}}_z(h_1)\right]^T d\xi e^{qt}dq$$

5. Optimization Model
Model 1 (concept model)

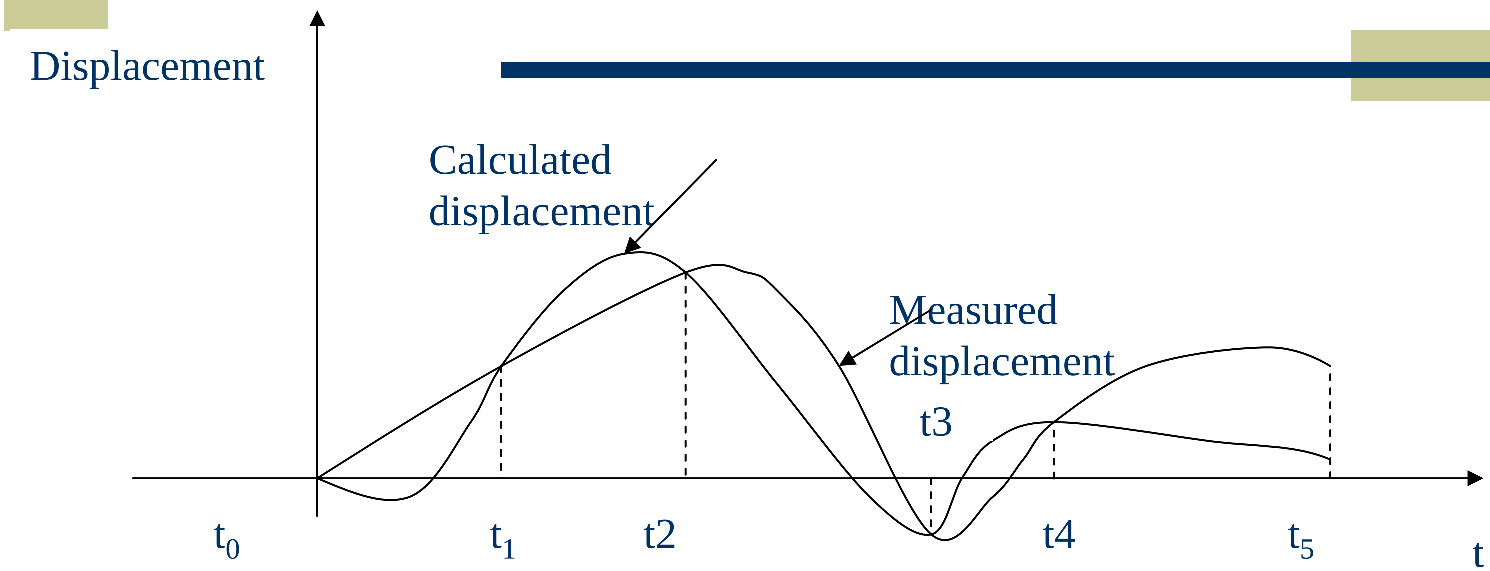


Displacement vs time

Min $\left(\frac{u^c(E_1, E_2, E_3) - u^m}{u^m}\right)^2$ s.t. $E_i^l \le E_i \le E_i^u$ i=1,2,3

Where u^c is calculated displacement, u^m measured displacement, E_i the modulus for ith layer, E_i^l the lower bound of modulus for ith layer, and E_i^u the upper bound of modulus for ith layer.

5. Optimization Model
Model 2 (mathematical model)

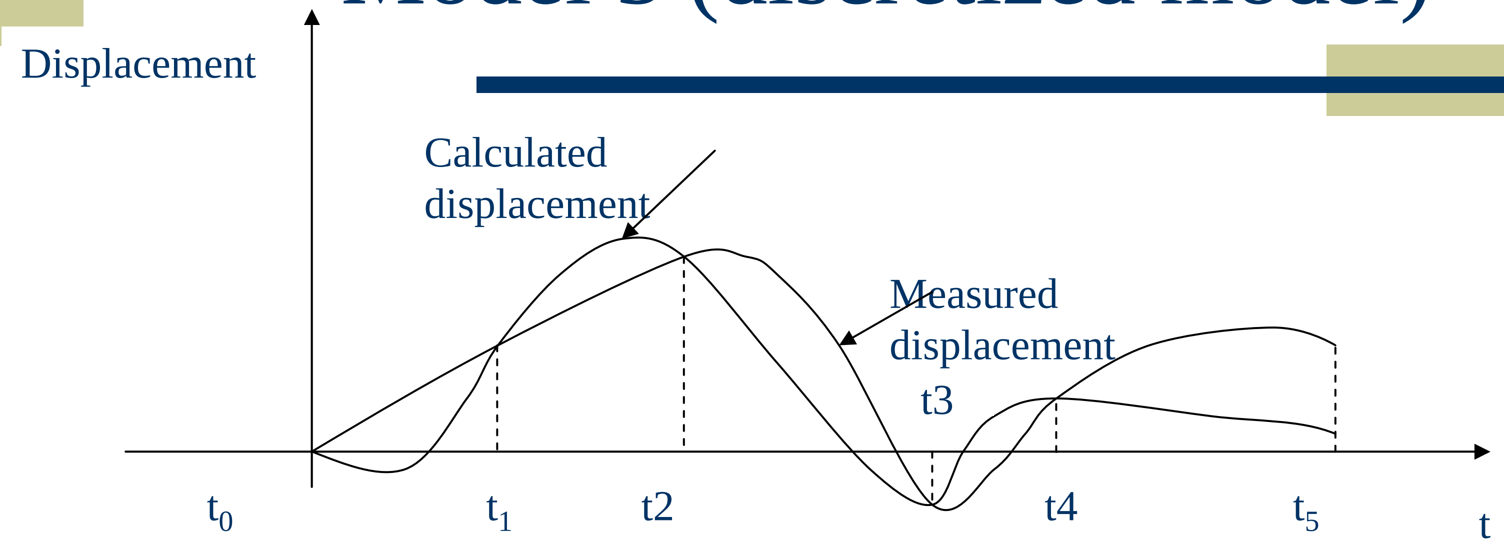


Displacement vs time

$$\text{Min} \sum_{n=0}^{k} \left(\int_{t_n}^{t_{n+1}} \frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} \, dt \right)^2$$

s.t. $E_i^l \le E_i \le E_i^u$ i=1,2,3

5. Optimization Model
Model 3 (discretized model)



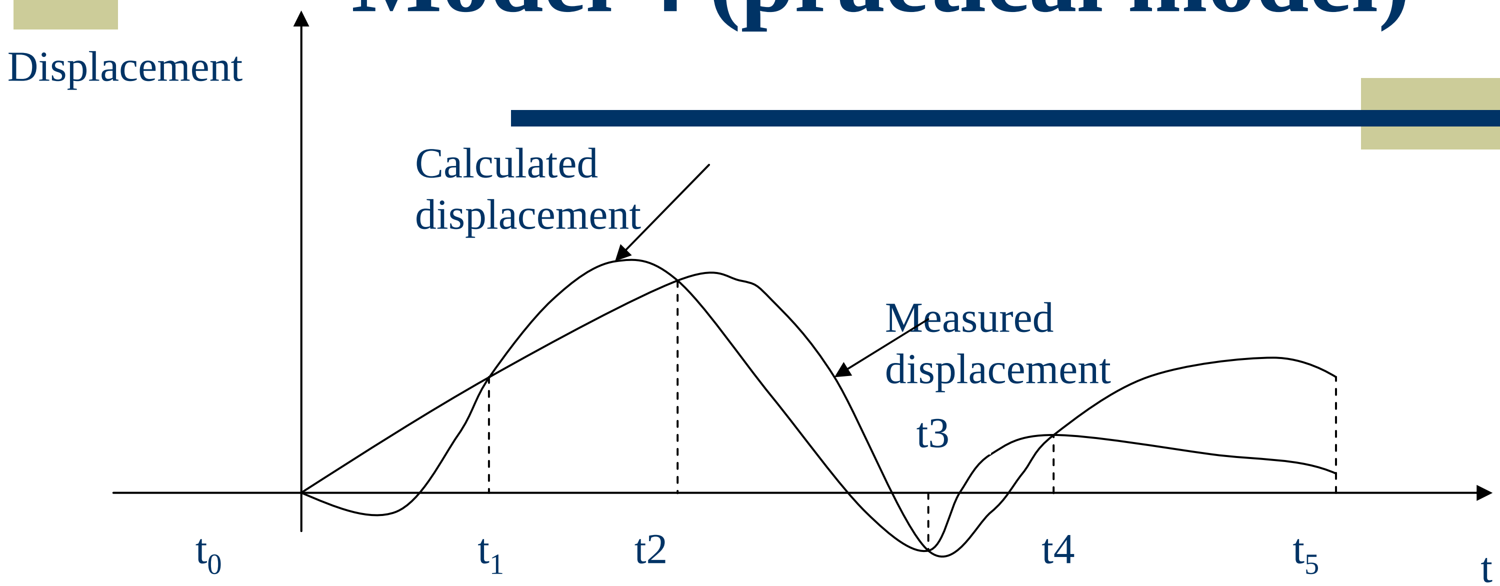
Displacement vs time

$$\text{Min} \sum_{n=0}^{N} \left(\int_{t_n}^{t_{n+1}} \frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} dt \right)^2$$

s.t. $E_i^l \le E_i \le E_i^u$ i=1,2,3

N>>k. $(t_{n+1}-t_n)$ is constant.

5. Optimization Model
Model 4 (practical model)



Displacement vs time

$$\text{Min} \quad \sum_{n=0}^{N} \left(\frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} \Delta t \right)^2$$

s.t. $E_i^l \leq E_i \leq E_i^u$ i=1,2,3

where $\Delta t = t_{n=1} - t_n$ is a constant.

6. Implementation Problem Statement

Min $\sum_{n=0}^{N} \left(\frac{u^c(E_1, E_2, E_3, t_n) - u^m(t_n)}{u^m(t_n)} \Delta t \right)^2$

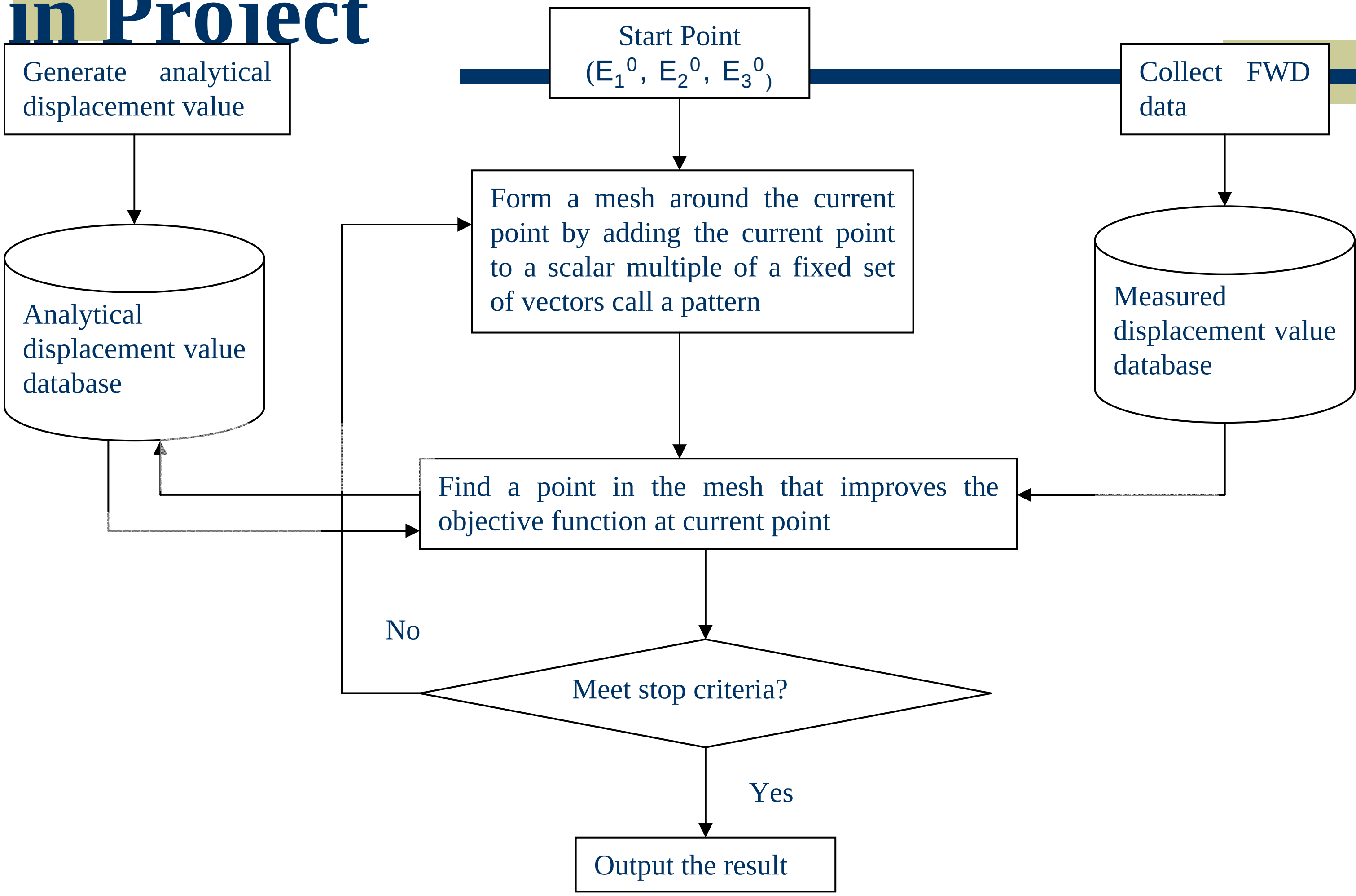
s.t. $E_i^l \leq E_i \leq E_i^u$ i=1,2,3

Where $u^c(E_1, E_2, E_3, t) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} \int_0^{\infty} \xi J_1(\xi r) [\psi_1(Z)] [\bar{\tilde{\tau}}_{rz}(h_1), \bar{\tilde{\sigma}}_z(h_1)]^T d\xi e^{qt} dq$

6. Implementation
Pattern Search Algorithm

- As we can see, the objective function is very complicated, and its gradient is not available. Direct search is a method for solving optimization problems that does not require any information about the gradient of the objective function. A special direct search method, pattern search algorithm is selected to solve the problem.
- Genetic Algorithm and Direct Search Toolbox in Matlab, is adopted to perform the optimization

6. Implementation
Application of Pattern Search Algorithm in Project



Flow Chart of Optimization Process

6. Implementation Optimization Result

- Start point (E_1^0, E_2^0, E_3^0) = (4.6200e+009 4.0270e+009 2.1240e+009)
- Optimal point (E_1^*, E_2^*, E_3^*) =(6.9871e+009 7.4255e+009 5.4802e+009)
- Optimal objective function $(\varepsilon^2)^* = 4.9432e-011$.

7. Conclusion

- Using time history of pavement responses to determine layer moduli leads to more precise outcome.

- Pattern search algorithm is proved to be valid and effective in backcalculation of pavement layer moduli.

8. Potential Limitation and Future Research

- The verification of the backcalculation using pattern search algorithm is not available in this project.
- In future research, several locations should be considered, and the weights allocated to different locations is to be studied.

Thank you!