Backcalculation Analysis of Pavement-layer Moduli Using Pattern Search Algorithms

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1. Introduction

- Evaluation of existing in-service pavement is important in determination of pavement construction quality and assessment of rehabilitation needs.

- Two possible methods for evaluating the pavement:
  (1) Lab testing
  (2) Nondestructive testing (NDT)
  Falling weight deflectometer (FWD)
2. Overview of the Project
Typical Flexible Pavement Structure

Asphalt Concrete

Base

Subbase/Subgrade
2. Overview of the Project
Falling Weight Deflectometer (FWD) Test

Schematic of Deflection Basin, and Loading, Sensors Configuration
3. Objective of the Project

- To develop an effective method to backcalculate the layer moduli of pavement from the FWD deflection data.
4. Pavement Model

Basic Model

\[ h_1 = 0 \]

\[ Z \]

Layer 1 \[ E_1, \nu_1, \rho_1 \] \[ \Delta h_1 \]

......

Layer i \[ E_i, \nu_i, \rho_i \] \[ \Delta h_i \]

......

Layer N \[ E_N, \nu_N, \rho_N \] \[ \Delta h_N \]

\[ h_{i+1} \]

\[ h_i \]

\[ h_{N+1} \]
4. Pavement Model

Governing Equation

\[
\begin{align*}
(c_d^2 - c_s^2) \frac{\partial \Delta}{\partial r} + c_s^2 \left( \nabla^2 u - \frac{u}{r^2} \right) - \ddot{u} &= 0 \\
(c_d^2 - c_s^2) \frac{\partial \Delta}{\partial z} + c_s^2 \nabla^2 w - \ddot{w} &= 0
\end{align*}
\]

- Where \( u = u(r,z,t) \): displacements of the \( i \) th layer along the \( r \);
- \( w = w(r,z,t) \): displacements of the \( i \) th layer \( z \) direction;
- dots indicate differentiation with respect to time \( t \);
- \( c_d \): dilatational wave velocity
- \( c_s \): shear wave velocity

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad \quad \Delta = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}
\]
4. Pavement Model Solution for Multiple layer Model with Rock bed.

\[ u(r, Z, t) = \frac{1}{2\pi i} \int_{\alpha-i\infty}^{\alpha+i\infty} \xi J_1(\xi r) \left[ y_1(Z) \left[ \overline{\sigma}_{rz} (h_1), \overline{\sigma}_z (h_1) \right]^T \right] d\xi d^q \]
5. Optimization Model

Model 1 (concept model)

\[
\text{Min} \left( \frac{u^c(E_1, E_2, E_3) - u^m}{u^m} \right)^2
\]

\[
\begin{align*}
\text{s.t.} & \quad E_i^l \leq E_i \leq E_i^u \\
& \quad i=1,2,3
\end{align*}
\]

Where \( u^c \) is calculated displacement, \( u^m \) measured displacement, \( E_i \) the modulus for \( i \) th layer, \( E_i^l \) the lower bound of modulus for \( i \) th layer, and \( E_i^u \) the upper bound of modulus for \( i \) th layer.
5. Optimization Model
Model 2 (mathematical model)

\[
\text{Min } \sum_{n=0}^{k} \left( \int_{t_n}^{t_{n+1}} \frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} \, dt \right)^2 \nonumber
\]

s.t. \( E_i^l \leq E_i \leq E_i^u \quad i=1,2,3 \)
5. Optimization Model

Model 3 (discretized model)

\[
\text{Min } \sum_{n=0}^{N} \left( \int_{t_n}^{t_{n+1}} \frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} dt \right)^2
\]

s.t. \( E_i^l \leq E_i \leq E_i^u \) \( i=1,2,3 \)

N>>k. \( (t_{n+1} - t_n) \) is constant.
5. Optimization Model
Model 4 (practical model)

\[
\text{Min} \quad \sum_{n=0}^{N} \left( \frac{u^c(E_1, E_2, E_3, t) - u^m(t)}{u^m(t)} \Delta t \right)^2
\]

s.t. \( E_i^l \leq E_i \leq E_i^u \) \( i=1,2,3 \)

where \( \Delta t = t_{n+1} - t_n \) is a constant.
6. Implementation

Problem Statement

\[
\text{Min} \quad \sum_{n=0}^{N} \left( u^c(E_1, E_2, E_3, t_n) - \frac{u^m(t_n)}{u^m(t_n)} \Delta t \right)^2
\]

s.t. \quad E_i^l \leq E_i \leq E_i^u \quad i=1,2,3

Where \( u^c(E_1, E_2, E_3, t) = \frac{1}{2\pi i} \int_{\alpha-\infty}^{\alpha+\infty} \xi J_1(\xi) \left[ \psi_1(Z) \right] ^T \tau_{r_z}(h_1), \sigma_z(h_1) \] d\xi^q d\eta
As we can see, the objective function is very complicated, and its gradient is not available. Direct search is a method for solving optimization problems that does not require any information about the gradient of the objective function. A special direct search method, pattern search algorithm is selected to solve the problem.

Genetic Algorithm and Direct Search Toolbox in Matlab, is adopted to perform the optimization
6. Implementation
Application of Pattern Search Algorithm in Project

Start Point $(E_1^0, E_2^0, E_3^0)$

Form a mesh around the current point by adding the current point to a scalar multiple of a fixed set of vectors call a pattern

Find a point in the mesh that improves the objective function at current point

Collect FWD data

Measured displacement value database

Generate analytical displacement value

Analytical displacement value database

Yes

No

Meet stop criteria?

Output the result

Flow Chart of Optimization Process
6. Implementation

Optimization Result

- Start point \((E_1^0, E_2^0, E_3^0) = (4.6200e+009, 4.0270e+009, 2.1240e+009)\)
- Optimal point \((E_1^*, E_2^*, E_3^*) = (6.9871e+009, 7.4255e+009, 5.4802e+009)\)
- Optimal objective function \((\varepsilon^2)^* = 4.9432e-011\)
7. Conclusion

- Using time history of pavement responses to determine layer moduli leads to more precise outcome.

- Pattern search algorithm is proved to be valid and effective in backcalculation of pavement layer moduli.
8. Potential Limitation and Future Research

- The verification of the backcalculation using pattern search algorithm is not available in this project.

- In future research, several locations should be considered, and the weights allocated to different locations is to be studied.
Thank you!