Decomposition Technique in Project Resource Planning Problem

ENCE 724 Nonlinear Programming/Spring 2005

Eakalak (Art) Intarakosit

Project Management Program

May 17, 2005

Decomposition Technique in Project Resource Planning Problem

Outline

1. Introduction
   - Project Network
2. Problem Statement and Objective
3. Model Formulation, Analysis
   - Benders Decomposition
4. Conclusion
5. Future Extension

INTRODUCTION

Project Network

1. Program Evaluation and Review Technique (PERT)

- Developed by U.S. Navy, Booz-Allen Hamilton, and Lockheed Aircraft (1950s)
- Used probabilistic estimates of activity durations

2. Critical Path Method (CPM)

- Developed by Dupont De nemours Inc.(1950s)
- Used deterministic estimates but included both time and cost estimates to allow time/cost trade-offs to be used
- Main purpose of CPM is to determine the “critical path”

INTRODUCTION

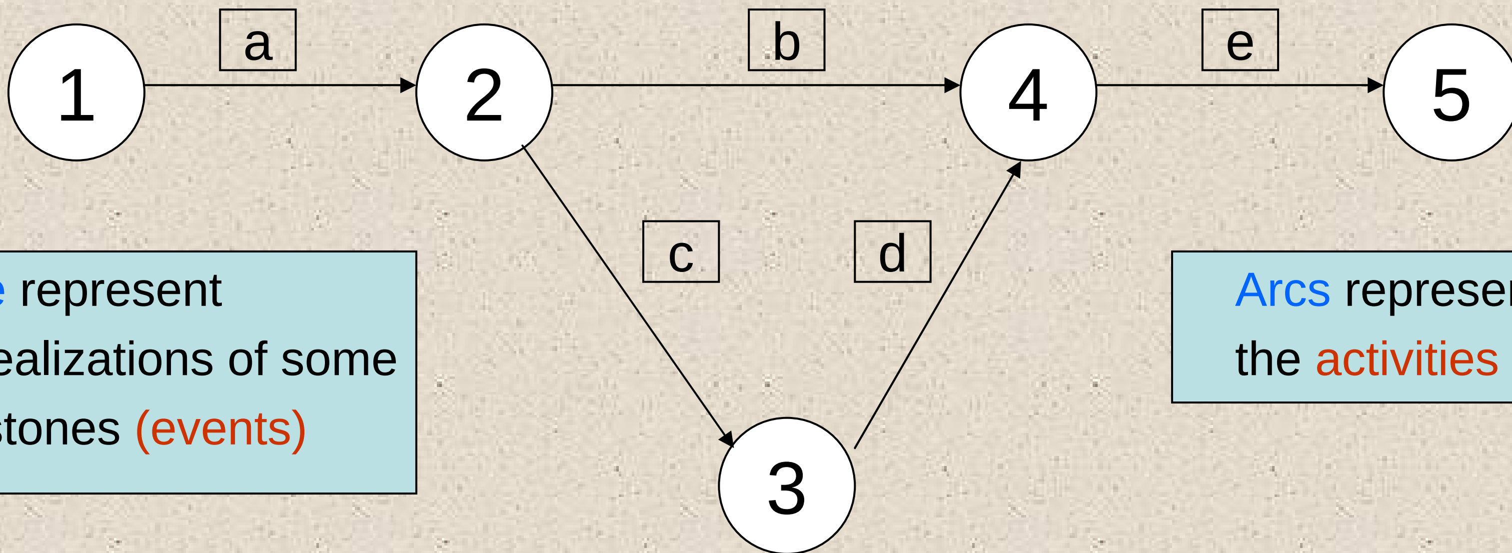
Displaying Project Network

1. Activity-on-arc (AOA)
   - Usually associated with PERT
   - Used in optimization setting (be used in the project)

2. Activity-on-node (AON)
   - Often associated with CPM
   - Used frequently in practical, non-optimization situations

INTRODUCTION

Example project network (AOA)



Node represent the realizations of some milestones (events)

Arcs represent the activities

PROBLEM STATEMENT

- The project itself have tasks, consume resources, and require time to complete the project
- The resources needed to perform the work can affect to both time and cost
- How to allocate resources to the project activity while minimizing time and cost of the project (Resource Allocation Problem)
- Objective

  - Defining and developing a solution approach for project resource planning

  - Focused on the way of defining relationships between resource and time allocated to tasks in order to minimize overall cost of project

MODEL FORMULATION

Min $\quad OH.t_n + \sum_{ij \in A} (c_{ij} r_{ij}) + \sum_{ij \in A} F_{ij}$

s.t.

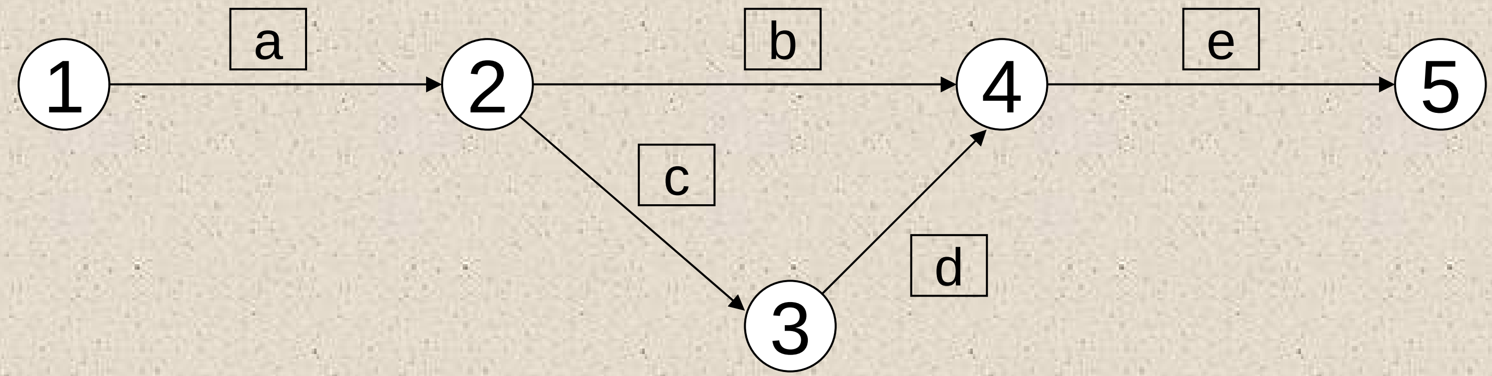
$$-t_i + t_j \geq d_{ij}(r_{ij}) \qquad \text{for all } (i,j) \in A \quad (1)$$

$$LB_{ij} \leq r_{ij} \leq UB_{ij} \qquad \text{for all } (i,j) \in A \quad (2)$$

$$t_i \geq 0$$

- Objective function attempts to minimize the overall cost of the project
- $\sum_{ij \in A} F_{ij}$ is a fixed cost associated with activity (i,j) which is not included in the model as it does not effect the optimal allocation
- Constraint (1) are the traditional precedence relations except that the activity's duration, $d_{ij}(r_{ij})$is now a function of the resource bundles allocated to the activity
- Constraint (2) establish the minimum resource requirements level (LB_{ij}), and the maximum resource allocation (UB_{ij})

MODEL FORMULATION



Activity	Node	LB(ij)	UB(ij)	c(ij)	$d_{ij}(r_{ij})$
a	(1,2)	1	3	10	2r12
b	(2,4)	1	4	20	4r24
c	(2,3)	1	3	10	2r23
d	(3,4)	1	4	15	4r34
e	(4,5)	1	5	10	5r45

BENDER DECOMPOSITION

Min $50t_5+10r_{12}+20r_{24}+10r_{23}+15r_{34}+10r_{45}$ (Cost)

s.t.
$$t_2-t_1 \geq 2r_{12}$$
$$t_4-t_2 \geq 4r_{13}$$
$$t_3-t_2 \geq 2r_{23}$$
$$t_4-t_3 \geq 4r_{34}$$
$$t_5-t_4 \geq 5r_{45}$$
$$1 \leq r_{12} \leq 3$$
$$1 \leq r_{24} \leq 3$$
$$1 \leq r_{23} \leq 3$$
$$1 \leq r_{34} \leq 3$$
$$1 \leq r_{45} \leq 3$$
$$t_1,t_2,t_3,t_4,t_5 \geq 0$$

Key Idea

Take coefficients out

COMPLICATING VARIABLES

t1	t2	t3	t4	t5	r12	r24	r23	r34	r45
-1	1				-2				
	-1		1			-4			
	-1	1					-2		
		-1	1					-4	
			-1	1					-5
					1				
					1				
						1			
						1			
							1		
							1		
								1	
								1	
									1
									1

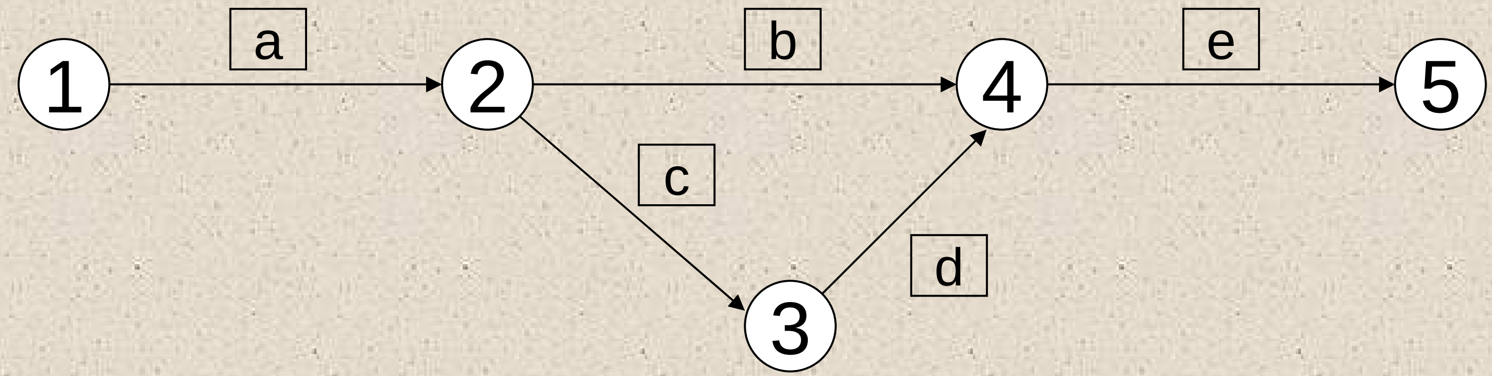
RESULTS

V	$r_{12}^{(V)}$	$r_{24}^{(V)}$	$r_{23}^{(V)}$	$r_{34}^{(V)}$	$r_{45}^{(V)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	2	4	8	13
2	1	1	1	1	1	0	2	4	8	13

V	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	100	0	100	200	250	715	15
2	650	100	0	100	200	250	715	715

ANALYSIS

The effect of changing $d_{ij}(r_{ij})$ – i.e. different expert opinion



Activity	Node	LB(ij)	UB(ij)	c(ij)	$d_{ij}(r_{ij})$
a	(1,2)	1	3	10	2r12+5
b	(2,4)	1	4	20	8r24
c	(2,3)	1	3	10	5r23
d	(3,4)	1	4	15	4r34+10
e	(4,5)	1	5	10	5r45

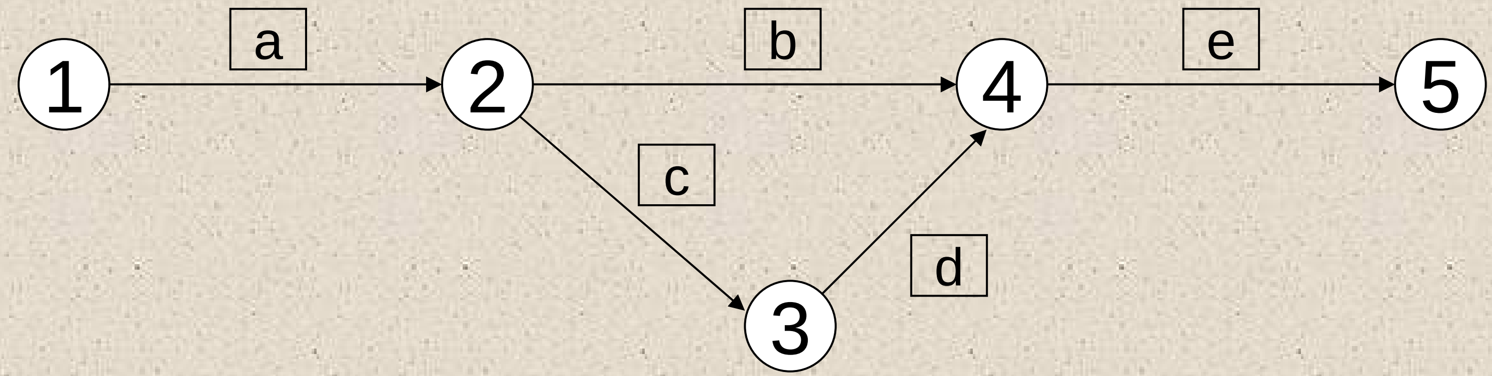
RESULTS

v	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	7	12	26	31
2	1	1	1	1	1	0	7	12	26	31

v	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	100	0	250	200	250	1615	15
2	650	100	0	100	200	250	1615	1615

ANALYSIS

The effect of changing $d_{ij}(r_{ij})$ – i.e. different expert opinion



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	1	3	10	10-2r12
b	(2,4)	1	4	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

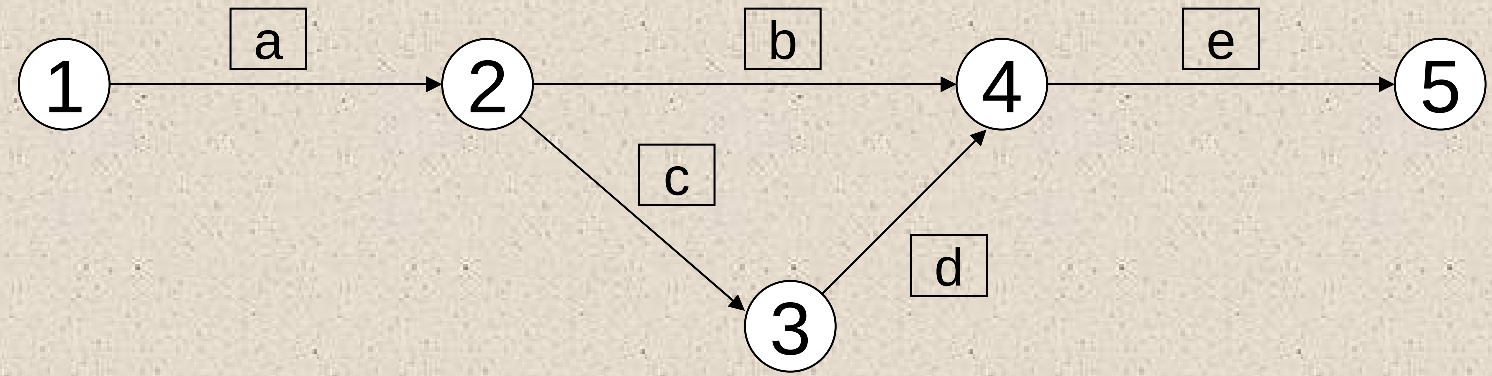
RESULTS

v	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	8	13	15	20
2	3	4	1	1	1	0	4	8	10	15
3	3	2	1	1	1	0	4	8	10	15

v	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	-50	0	0	250	1065	15
2	650	-100	0	200	100	250	895	795
3	750	-100	0	200	100	250	855	855

ANALYSIS

The effect of minimum resources needed to perform the work



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	2	3	10	10-2r12
b	(2,4)	1	4	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

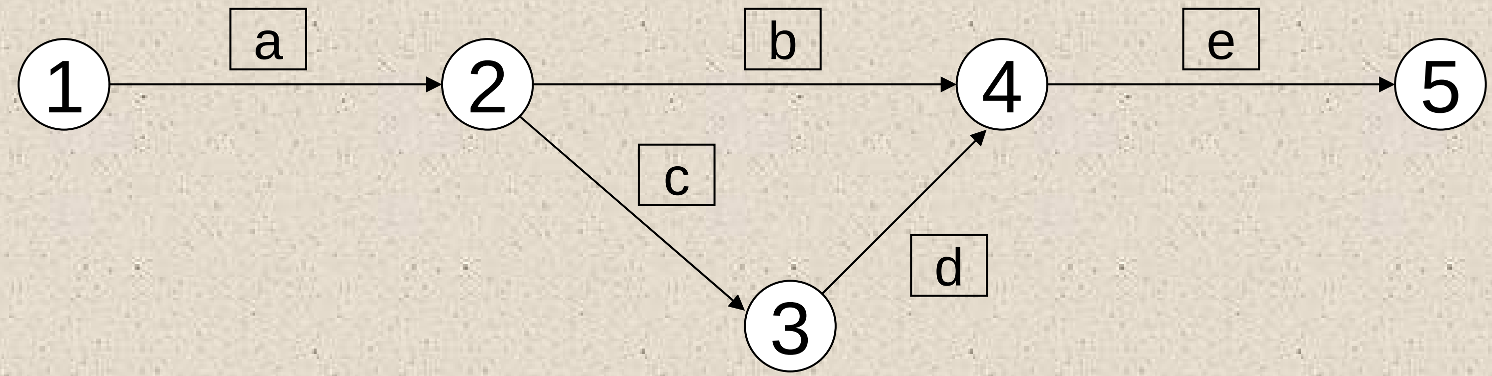
RESULTS

V	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	2	1	1	1	1	0	6	11	13	18
2	3	4	1	1	1	0	4	8	10	15
3	3	2	1	1	1	0	4	8	10	15

V	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	-50	0	0	250	975	25
2	650	-100	0	200	100	250	895	795
3	750	-100	0	200	100	250	855	855

ANALYSIS

The effect of increasing maximum resources



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	1	4	10	10-2r12
b	(2,4)	1	4	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

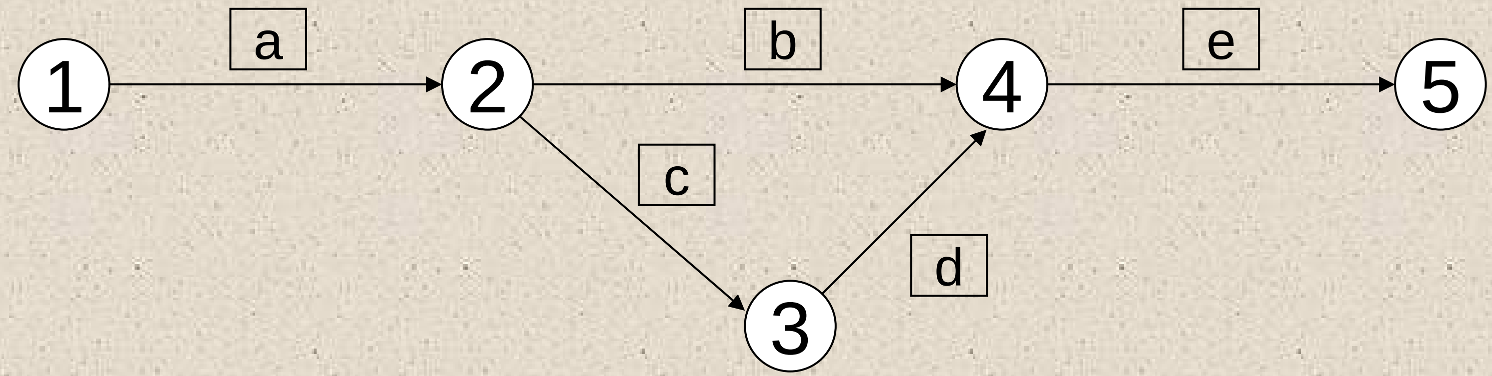
RESULTS

v	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	8	13	15	20
2	4	4	1	1	1	0	0	4	6	13
3	4	2	1	1	1	0	2	6	8	13

v	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	-50	0	0	250	1065	15
2	550	-100	0	200	100	250	805	705
3	650	-100	0	200	100	250	765	765

ANALYSIS

The effect of increasing maximum resources



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	1	5	10	10-2r12
b	(2,4)	1	4	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

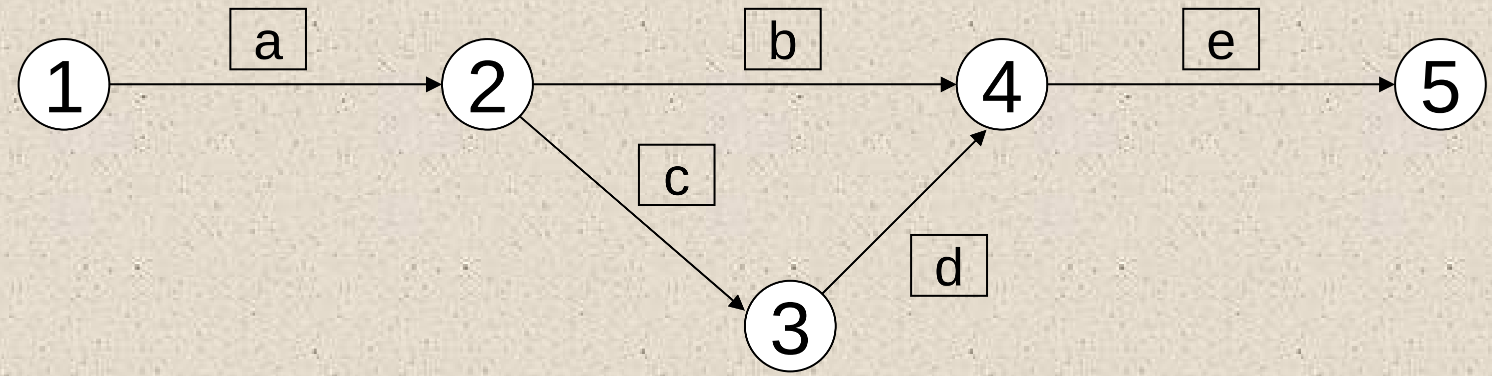
RESULTS

v	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	8	13	15	20
2	5	4	1	1	1	0	0	4	6	11
3	5	2	1	1	1	0	0	4	6	11

v	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	-50	0	0	250	1065	15
2	450	0	0	200	100	250	715	615
3	550	0	0	200	100	250	675	675

ANALYSIS

The effect of minimum resources needed to perform the work



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	1	3	10	10-2r12
b	(2,4)	3	4	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

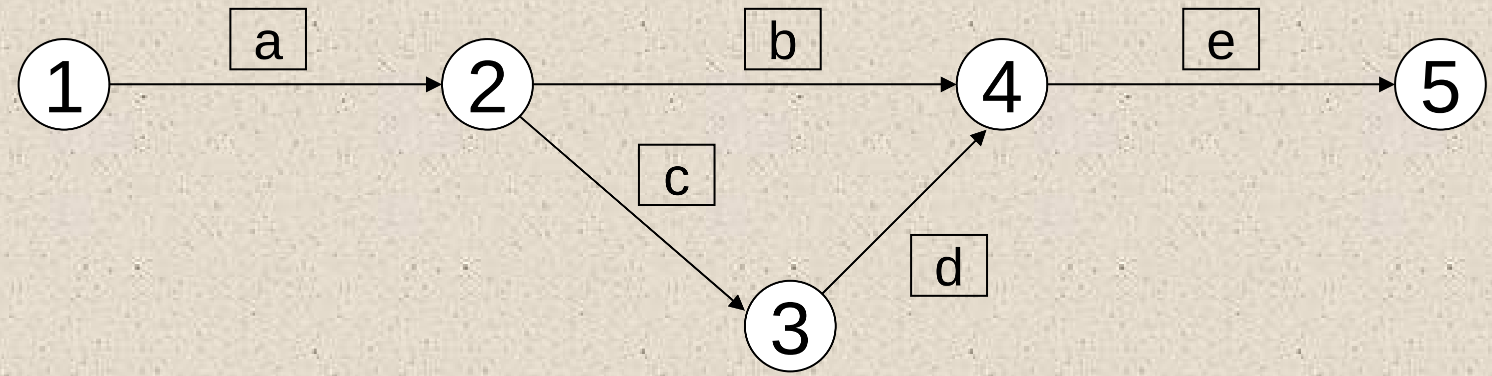
RESULTS

V	$r_{12}^{(V)}$	$r_{24}^{(V)}$	$r_{23}^{(V)}$	$r_{34}^{(V)}$	$r_{45}^{(V)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	3	1	1	1	0	8	12	14	19
2	3	3	1	1	1	0	4	8	10	15

V	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	0	200	100	250	1055	55
2	750	-100	0	200	100	250	875	875

ANALYSIS

The effect of increasing maximum resources



Activity	Node	LB(ij)	UB(ij)	c(ij)	dij(rij)
a	(1,2)	1	3	10	10-2r12
b	(2,4)	1	5	20	8-r24
c	(2,3)	1	3	10	4r23
d	(3,4)	1	4	15	2r34
e	(4,5)	1	5	10	5r45

RESULTS

V	$r_{12}^{(v)}$	$r_{24}^{(v)}$	$r_{23}^{(v)}$	$r_{34}^{(v)}$	$r_{45}^{(v)}$	$t_1^{(v)}$	$t_2^{(v)}$	$t_3^{(v)}$	$t_4^{(v)}$	$t_5^{(v)}$
1	1	1	1	1	1	0	8	13	15	20
2	3	5	1	1	1	0	4	8	10	15
3	3	2	1	1	1	0	4	8	10	15

V	$\alpha^{(v)}$	λ_{12}	λ_{24}	λ_{23}	λ_{34}	λ_{45}	$z_{(v)}^{up}$	$z_{(v)}^{down}$
1	-50	-100	-50	0	0	250	1065	15
2	600	-100	0	200	100	250	915	765
3	750	-100	0	200	100	250	855	855

CONCLUSIONS

- Benders decomposition approach to project resource planning
- Express the activity's duration as a function of the resource allocated to the activity, $d_{ij}(r_{ij})$
- The factors that effect the overall cost

  - Relationship of resource and time (expert opinion, intuitive)

  - The minimum and maximum of resource
- LINDO is also used to check the solution

FUTURE EXTENSION

- Benders decomposition would be used for thesis in the topic of Project Topology Network
- Network is more complicated, i.e. 100 activities
- Find an optimal network that minimize time and cost while also maximize the scope of the project

Decomposition Technique in Project Resource Planning Problem

QUESTIONS