An Optimal Transmitting Scheme for MIMO-OFDM Systems with Spatial Correlation Feedback

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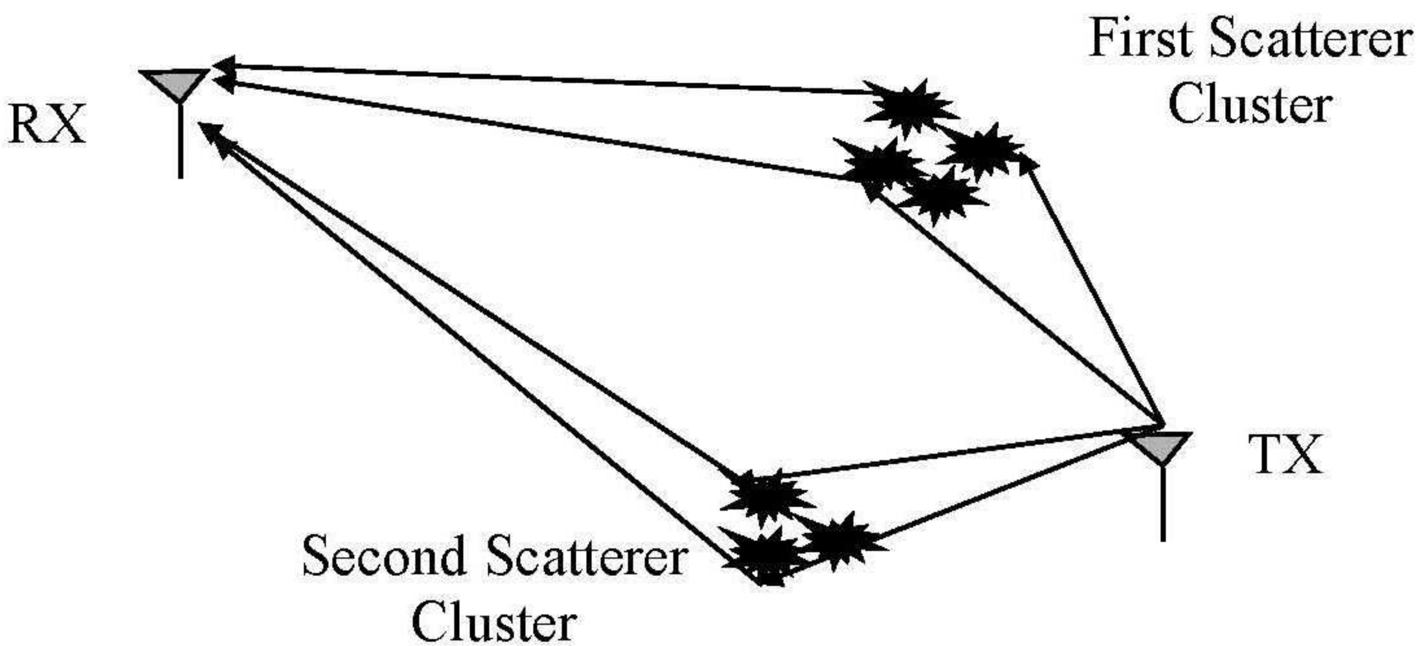

Different Scatterers are independent.

Motivation

- Degrees of Channel State Information at the transmitter:
  -Perfect CSI (unpractical)
  -Mean Feedback (applications with low mobility)
  -Covariance Feedback (applications that involve mobile transmitters and/or receivers)

- The channel model that is widely used: Flat Fading Scenario which models narrowband applications (Speech)

- Frequency Selective Fading: Models broadband applications (high data rate, multimedia applications , and others) Only Perfect CSI: (Cioffi et. al., SP 03) and Mean Feedback: (Giannakis et. al., SP 04)

System Model

- MIMO-OFDM frequency selective fading channel model with M_t transmit antennas and M_r receive antennas.

- The channel impulse response from transmit antenna i to receive antenna j at any time instant τ can be modeled as

$$h_{ij}(\tau) = \sum_{l=0}^{L-1} \alpha_{ij}(l)\,\delta(\tau - \tau_l)\,, \qquad (1)$$

- The received signal at the n-th subcarrier at receive antenna j is given by

$$y_j(n) = \sqrt{\frac{\rho}{M_t}}\mathbf{h}_j^T(n)\,\mathbf{b}(n) + v_j(n)\,, \qquad (2)$$

System Model

- In general, the design of the SF-beamformer symbol $\mathbf{B}$ is split into two parts $\mathbf{B} = \mathbf{W}\mathbf{C}$.
- A predesigned SF codeword $\mathbf{C}$
- A beamformer (linear transformation) $\mathbf{W}$ which is an $M_t \times M_t$ matrix
- We seek the beamformer design $\mathbf{W}$!!!

- Objective: Minimize the average pairwise error probability

$$Pr(\mathbf{B} \rightarrow \tilde{\mathbf{B}}) \tag{3}$$

- We can prove that the average pairwise error probability can be upper bounded by

$$PEP \leq \left[\det\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H\left(I_{NM_r} \otimes \left[(I_N \otimes \mathbf{W})\, \mathbf{\Delta} \left(I_N \otimes \mathbf{W}^H\right)\right]\right)\tilde{\mathbf{R}} + I_r\right)\right]^{-} \tag{4}$$

- Where $\tilde{\mathbf{R}}$ is the square root matrix of the channel correlation matrix $\mathbf{R_h} = E\left\{\mathbf{hh}^H\right\}$

$$\mathbf{R_h} = \tilde{\mathbf{R}}\tilde{\mathbf{R}}^H \tag{5}$$

Nonlinear Programming Concepts

- We can state the optimization problem as follows

$$\max_{\mathbf{W}} f(\mathbf{W}) = \det\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H \left(I_{NM_r} \otimes \left[(I_N \otimes \mathbf{W})\,\Delta\left(I_N \otimes \mathbf{W}^H\right)\right]\right)\tilde{\mathbf{R}} + I_r\right)$$
$$\text{s.t. } \|\mathbf{W}\|_F^2 = 1. \quad (6)$$

- Is it convex? (Hopefully!)

- Δ is a positive semi-definite Hermitian matrix that depends on the SF code design

- Relaxation: Assume the matrix Δ to be identity!

- The optimization problem then reduces to

$$\max_{\mathbf{W}} f(\mathbf{W}) = \text{det}\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H \left(I_{NM_r} \otimes \left[(I_N \otimes \mathbf{W})\left(I_N \otimes \mathbf{W}^H\right)\right]\right)\tilde{\mathbf{R}} + I_r\right)$$
$$\text{s.t. } \|\, \mathbf{W} \,\|_F^2 = 1. \tag{7}$$

- Define $\eta = \mathbf{W}\mathbf{W}^H$.
Matrix identity $(I_N \otimes A)(I_N \otimes A^H) = I_N \otimes (AA^H)$

- The optimization problem can be rewritten as follows

$$\max_{\eta} g(\eta) = \text{det}\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H \left(I_{NM_r} \otimes (I_N \otimes \eta)\right)\tilde{\mathbf{R}} + I_r\right)$$
$$\text{s.t. } \text{tr}(\eta) = 1,$$
$$\eta = \eta^H \geq 0. \tag{8}$$

- Using the matrix identity $I_N \otimes (I_M \otimes A) = I_{NM} \otimes A$

- The objective function can be finally written as

$$\begin{aligned} \max_{\eta}\ g(\eta) &= \det\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H \left(I_{N^2M_r} \otimes \eta\right)\tilde{\mathbf{R}} + I_r\right) \\ \text{s.t. } \operatorname{tr}(\eta) &= 1, \\ \eta &= \eta^H \geq 0. \end{aligned} \tag{9}$$

- First we study the convexity of the feasible region
  Let $\eta = \lambda\eta_1 + (1-\lambda)\eta_2$, where $\lambda \in (0,1)$

- Then we have

$$\begin{aligned} \operatorname{tr}(\eta) =& \operatorname{tr}(\lambda\eta_1 + (1-\lambda)\eta_2) = \lambda\operatorname{tr}(\eta_1) + (1-\lambda)\operatorname{tr}(\eta_2) \\ &= \lambda + (1-\lambda) = 1. \end{aligned} \tag{10}$$

- Substituting $\eta = \lambda\eta_1 + (1-\lambda)\eta_2$ in $g(\eta)$

$$g(\eta) = \det\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H\left(I_{N^2M_r} \otimes (\lambda\eta_{\mathbf{1}} + (\mathbf{1}-\lambda)\eta_{\mathbf{2}})\right) \times\tilde{\mathbf{R}} + I_r\right)$$

$$\det\left(\lambda\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H\left(I_{N^2M_r} \otimes \eta_1\right)\tilde{\mathbf{R}} + (1-\lambda)\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H\left(I_{N^2M_r} \otimes \eta_2\right)\right. \quad (11)$$

- **Corollary (Matrix Analysis)** For any positive definite matrices A and B and $\lambda \in (0,1)$, the following is true

$$\det[\lambda A + (1-\lambda)B] \geq \det[A]^{\lambda}\det[B]^{1-\lambda}. \quad (12)$$

- Using this corollary, we get

$$g(\eta) \geq g(\eta_1)^{\lambda} g(\eta_2)^{1-\lambda}. \quad (13)$$

- Define $q(\eta) = \log(g(\eta))$, then the above equation reduces to

$$q(\eta) \geq \lambda q(\eta_1) + (1 - \lambda) q(\eta_2). \tag{14}$$

- Therefore $q(\eta)$ is a concave function!

- Now we can write the optimization problem again as follows

$$\begin{aligned} \max_{\eta} \; & q(\eta) = \log\left(\det\left(\frac{\rho}{4M_t}\tilde{\mathbf{R}}^H \left(I_{N^2 M_r} \otimes \eta\right)\tilde{\mathbf{R}} + I_r\right)\right) \\ \text{s.t. } & \operatorname{tr}(\eta) = 1, \\ & \eta = \eta^H \geq 0. \end{aligned} \tag{15}$$

- This is maximizing a concave function over a convex set Local optima are global optima !

Numerical Solution

- To solve the optimization problem we choose the sequential quadratic programming (SQP) method.

- Consider solving the following problem

$$\begin{aligned} &\min_x f(x) \\ &g_i(x) = 0, \quad 1 \leq i \leq m_e \\ &g_i(x) \leq 0, \quad m_e + 1 \leq i \leq m, \\ &x \in \mathcal{R}^n. \end{aligned} \tag{16}$$

- For a given iteration k, let x_k be an approximation to the solution, λ_k be an approximation to the multipliers, and B_k an approximation to the Hessian matrix of the Lagrangian

- we have to solve the following quadratic programming problem

$$\begin{aligned} &\min_{d} \frac{1}{2} d^T B_k d + \nabla f(x_k)^T d \\ &\text{s.t. } \nabla g_i(x_k)^t d + g_i(x_k) = 0, \quad 1 \le i \le m_e, \\ &\quad \nabla g_i(x_k)^t d + g_i(x_k) \le 0, \quad m_e + 1 \le i \le m, \end{aligned} \tag{17}$$

- Let d_k be the optimal solution of the above subproblem and μ_k the corresponding multipliers, then the next iteration is formulated as follows

$$\binom{x_{k+1}}{\lambda_{k+1}} = \binom{x_k}{\lambda_k} + \alpha_k \binom{d_k}{\mu_k - \lambda_k} \tag{18}$$

- SQP is implemented in the Matlab optimization toolbox by the function 'fmincon'

Numerical Solution

- In the simulation experiments, the square root of the channel covariance matrix was taken to be

$$\tilde{\mathbf{R}}^H = \left[\begin{array}{ccccccc} -0.6918 & 1.2540 & -1.4410 & -0.3999 & 0.8156 & 1.2902 & 1 \\ -1.5937 & 0.5711 & 0.6900 & 0.7119 & 0.6686 & -1.2025 & - \end{array} \right. \quad (19)$$

- The matrix η is a 2×2 symmetric matrix, i.e. 3 unknowns.

- The cost function can be simplified as follows

$$\begin{aligned} q(\eta) = \log(&13.57 * \eta(1,1)^2 - 5.8645 * \eta(1,2)^2 + 9.88 * \eta(2,2)^2 - \eta(\\ &+ 28 * \eta(1,1) * \eta(2,1) + 10.75 * \eta(1,2) * \eta(2,1) + (4/SNR) \\ &+ 0.8572 * \eta(1,2) + 6.88 * \eta(2,1) + 1)). \end{aligned} \quad (20)$$

- We tested the algorithm at more than initial point and they all gave the same result as follows

$$\begin{aligned} x_o &= [0.5, 0, 0.5]; \\ x_o &= [1, 0, 0]; \\ x_o &= [0, 0, 1], \\ x_o &= [0.2, 0, 0]. \end{aligned} \tag{21}$$

  The optimal solution at any of the above initial conditions was

$$\eta_1(1,1) = 0.3416, \quad \eta_1(1,2) = 0.4742, \quad \eta_1(2,2) = 0.6584. \tag{22}$$

- The above answer was at $SNR = 100$. We changed the SNR to check the sensitivity of the optimal solution to changing the SNR.

- We considered $SNR = 10$ and the results were

$$\eta_2(1,1) = 0.3614, \quad \eta_2(1,2) = 0.4804, \quad \eta_2(2,2) = 0.6386. \tag{23}$$

- Thus the optimal solution is not sensitive to the variations in the SNR.

- Although this phenomenon was only checked for this specific example, it is intuitive to generalize it.

Conclusions and Future work

- We studied the problem of optimal transmitter design in the presence of partial channel state information

- We formulated the problem as a non-linear optimization problem where the objective function is an upper-bound on the system pairwise error probability and the constraints are on the system energy

- Under a relaxation on the objective function, we prove that the optimization problem is convex

- We utilize the sequential quadratic programming methods to solve the problem

- Future Work