Resource Allocation in OFDMA Wireless Networks

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Term Project

Outline

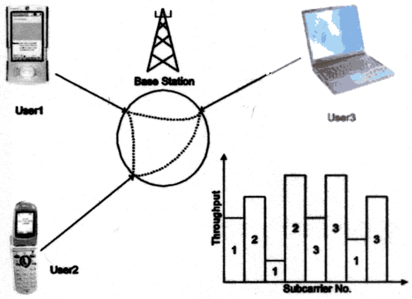
- Backgrounds
- Resource allocation in OFDMA systems
  - Adaptive bit, power, subchannel allocation
  - Problem formulation
  - Optimization for two-user allocation
  - Optimization for multiple-user allocation
- Simulation results
- Conclusion

Backgrounds

- Wireless channels
  - Doppler frequency shift (time selective fading)
  - Multipath effect (frequency selective fading)
- Wireless communication systems
  - TDMA (GSM)
  - CDMA (3Gpp)
  - OFDMA (Orthogonal Frequency Division Multiple Access): Wireless LAN Applications
- Resource allocation
  - Power control and adaptive modulation
  - Sub-channels
    (frequency sub-band, time slot, spreading code)

System Description

- Single-cell uplink OFDMA systems
  - N subchannels, K users
- Optimization Goal
  - Maximize the overall rates
- Competition: the same subcarrier may be good for many users. But only one user per channel.
- Constraints
  - Maximal transmitted power
  - Minimal requirement rate



Resource Allocation in OFDMA Systems

- System model
  - Define the rate allocation matrix **r** as $[\mathbf{r}]_{ij} = r_{ij}$
  - Define the subcarrier assignment matrix $[\mathbf{A}]_{ij} = a_{ij}$, where $\sum_{i=1}^{K} a_{ij} = 1, \forall j$ and
    $$a_{ij} = \begin{cases} 1, & \text{if } r_{ij} > 0; \\ 0, & \text{otherwise.} \end{cases}$$
  - Define the power allocation matrix **P** as $[\mathbf{P}]_{ij} = P_{ij}$.
- Adaptive modulation
  - Variable bits can loaded to the subcarrier;
  - Higher modulation level requires higher power.

$$r_{ij} = W \log_2 \left(1 + \frac{P_{ij} G_{ij} c_3}{\sigma^2}\right)$$

The optimization problem

- The optimization goal

$$\max_{\mathbf{A},\mathbf{P}} U$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{K} a_{ij} = 1, \forall j; \\ R_i \geq R^i_{min}, \forall i; \\ \sum_{j=1}^{N} P_{ij} \leq P_{max}, \forall i. \end{cases}$$

$$\text{Maximal Rate} : U = \sum_{i=1}^{K} R_i,$$

$$\text{Max-min Fairness} : U = \min R_i,$$

$$\text{Proportional Fairness} : U = \Pi_{i=1}^{K} (R_i - R^i_{min}).$$

* Nonlinear non-concave integer programming;

* Simplification method:

Two-user case, continuous relaxation.

Two-user Resource Allocation

- Resource allocation with fixed channel allocation (well-known water-filling algorithm)

$$\max_{\mathbf{P}} U_i = R_i$$

$$\text{subject to } \sum_{j=1}^{N} P_{ij} a_{ij} \le P_{max}, \forall i.$$

$$P_{ij} = (\mu_i - I_{ij})^+ \text{ and } r_{ij} = W \log_2(1 + \frac{P_{ij} a_{ij}}{I_{ij}}), \quad I_{ij} = \frac{\sigma^2}{c_3 G_{ij}}$$

- Lagrange function for the two-user case

$$L = \left(\sum_{j=1}^{N} a_{1j} W \log_2\left(1 + \frac{P_{1j} G_{1j} c_3}{a_{1j}\sigma^2}\right) - R_{min}^1\right)\left(\sum_{j=1}^{N} a_{2j} W \log_2\left(1 + \frac{P_{2j} G_{2j} c_3}{a_{2j}\sigma^2}\right) - R_{min}^2\right)$$

$$+ \sum_{j=1}^{N} \lambda_j \left(\sum_{i=1}^{2} a_{ij} - 1\right) + \sum_{i=1}^{2} \kappa_i \left(\sum_{j=1}^{N} P_{ij} - P_{max}\right) - \sum_{i=1}^{2}\sum_{j=1}^{N} \nu_{ij}^1 P_{ij} - \sum_{i=1}^{2}\sum_{j=1}^{N} \nu_{ij}^2 a_{ij},$$

Two-band Partition Algorithm (1)

TWO-USER ALGORITHM

1. Initialization: Initialize the subcarrier assignment with the minimal rate requirements. For *Maximal Rate*, $\varrho_1 = \varrho_2 = 1$; For *NBS*, calculate ϱ_1 and ϱ_2.
2. Sort the subcarriers: Arrange the index from the largest to smallest $\frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}}$.
3. For j=1,. . . , N-1 User 1 occupies and water-fills subcarrier 1 to j; User 2 occupies and water-fills subcarrier j+1 to N. Calculate U. **End**
4. Choose the two-band partition (the corresponding j) that generates the largest U satisfing the constraints. Calculate **A**, **P**, R_1, and R_2.
5. Update channel assignment -*Maximal Rate*: Return -*NBS*: If U can not be increased by updating ϱ_1 and ϱ_2, the iteration ends; otherwise, update $\varrho_1 = 1/(R_1 - R_{min}^1)$, $\varrho_2 = 1/(R_2 - R_{min}^2)$; go to Step 2.

Two-band Partition Algorithm (2)

- **Proof. (scratch)**

$$\frac{\log_2\left(1+\frac{P_{1j}G_{1j}c_3}{a_{1j}\sigma^2}\right)-\frac{\frac{P_{1j}G_{1j}c_3}{a_{1j}\sigma^2}}{1+\frac{P_{1j}G_{1j}c_3}{a_{1j}\sigma^2}}}{\left(\sum_{j=1}^{N} a_{1j}W\log_2\left(1+\frac{P_{1j}G_{1j}c_3}{a_{1j}\sigma^2}\right)-R_{min}^1\right)}=\frac{\log_2\left(1+\frac{P_{2j}G_{2j}c_3}{a_{2j}\sigma^2}\right)-\frac{\frac{P_{2j}G_{2j}c_3}{a_{2j}\sigma^2}}{1+\frac{P_{2j}G_{2j}c_3}{a_{2j}\sigma^2}}}{\left(\sum_{j=1}^{N} a_{2j}W\log_2\left(1+\frac{P_{2j}G_{2j}c_3}{a_{2j}\sigma^2}\right)-R_{min}^2\right)}.$$

$$\varrho_i=\begin{cases} 1/\left(\sum_{j=1}^{N} a_{ij}W\log_2\left(1+\frac{P_{ij}G_{ij}c_3}{a_{ij}\sigma^2}\right)-R_{min}^i\right), & \sum_{j=1}^{N} a_{ij}W\log_2\left(1+\frac{P_{ij}G_{ij}c_3}{a_{ij}\sigma^2}\right)\geq R_{min}^i+\epsilon;\\ 1/\varepsilon, & \text{otherwise,}\end{cases}$$

$$\varrho_1\left(\log_2\left(\frac{\mu_1}{I_{1j}}\right)+\frac{I_{1j}}{\mu_1}-1\right)=\varrho_2\left(\log_2\left(\frac{\mu_2}{I_{2j}}\right)+\frac{I_{2j}}{\mu_2}-1\right).$$

$$f\left(\frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}}\right)\approx\log_2\left(\frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}}\right)+\log_2\left(\frac{\mu_1^{\varrho_1}}{\mu_2^{\varrho_2}}\right)+\varrho_2-\varrho_1.\quad \text{Let } g_{ij}=1/I_{ij}.$$

Multiple-user Resource Allocation

MULTIUSER ALGORITHM

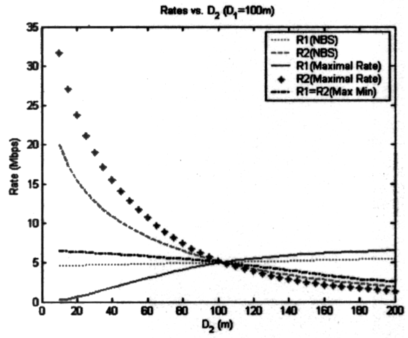
1. Initialize the channel assignment: Assign all subcarriers to users.
2. Coalition Grouping: If the number of users is even, the users are grouped into coalitions; otherwise, a dummy user is created to make the total number of users even. No user can exchange its resource with this dummy user. - *Random Method*: Randomly form 2-user coalition. - *Hungarian Method*: Form user coalitions by the Hungarian algorithm.
3. Bargain within Each Coalition: Negotiate between two users in all coalitions to exchange the subcarriers using the two-user algorithm in Table I.
4. Repeat: Repeat Step 2 and Step 3, until no further improvement can be achieved.

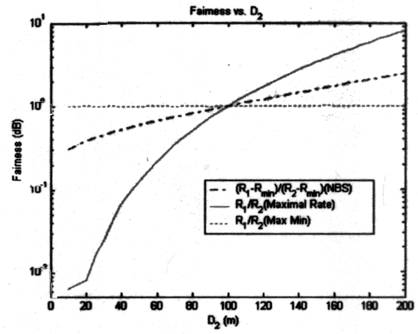
Hungarian Method

B \ G	H	I	J	K	L
A	1	2	3	4	5
B	2	3	1	5	4
C	3	5	1	2	4
D	1	3	2	4	5
E	4	2	5	1	3

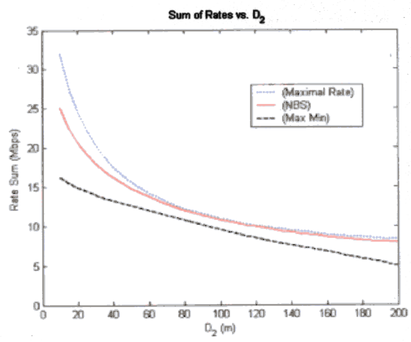
B \ G	H	I	J	K	L
A	0	1	0	0	0
B	0	0	0	0	1
C	0	0	1	0	0
D	1	0	0	0	0
E	0	0	0	1	0

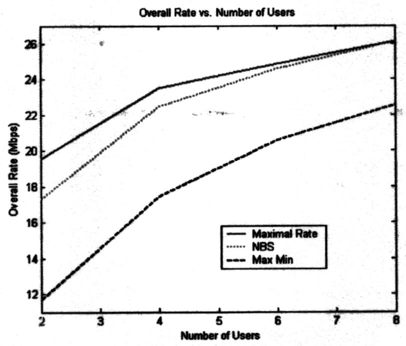
Simulation Results (1)



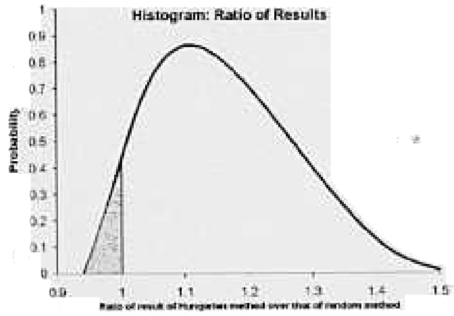


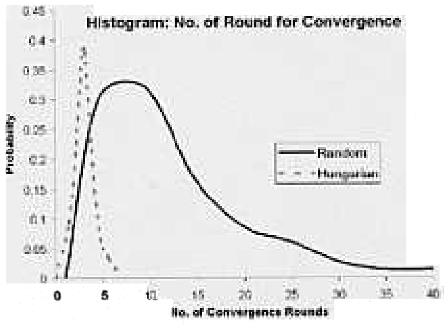
Simulation Results (2)





Simulation Results (3)





Conclusions

- Resource allocation in OFDMA systems is modeled as a nonlinear programming problem.
- KKT condition is utilized to develop the algorithm for the integer programming problem.
- Multiple-user programming is simplified by two-user algorithm and pairing method.
- The simulation results compares the fairness property for different objective functions.